SOLUȚII

ALGEBRĂ

CAPITOLUL 1
Calculul algebric

1.1. Adunarea şi scăderea numerelor reale reprezentate prin litere

1. a) $-2xy$; b) $-0{,}6x^2$; c) $3\sqrt{3}$; d) $-\frac{5}{3}b$; e) $\sqrt{2}\,x$; f) $-0{,}1xyz$; g) a^3b; h) $-\frac{\sqrt{2}}{2}a^2b^2$. **2.** a) $-5a$; b) $-3a$; c) $19x$; d) $-b$. **3.** a) $5x$; b) $-2{,}4b$; c) $9x$; d) $4{,}2x$. **4.** a) $19x^2$; b) $-5y^3$; c) $1{,}8b^4$; d) $2x^3$; **5.** a) $a^2b + 9ab^2$; b) $3x^2 - 3y^2 + 7xy$; c) $3{,}8x^2y^2 - 2{,}1xy^2 + 3{,}3x^2y$; d) $-9x + 6y + 29$. **6.** a) $x\left(3\sqrt{2}+8\right)+y\left(2\sqrt{3}+9\right)$; b) $8y\left(3+\sqrt{5}-\sqrt{3}\right)$; c) $4\sqrt{7}y-15x$; d) 0. **7.** a) $8x$; b) $3a$; c) $-8y^2$; d) $-9xy$. **8.** a) $N = a + b$ cu $a = 5x^2$, $b = 4y$; b) $N = a + b + c$ cu $a = 5x^2$, $b = y$, $c = 3y$; c) $N = a - b$ cu $a = 5x^2$, $b = -4y$. **9.** a) $-\frac{1}{9}ab$; b) $2y - b$; c) $x - y$; d) $2a$. **10.** a) $x\sqrt{3}$; b) $28\sqrt{2}\,b$; c) $ab\left(14\sqrt{7}-5\sqrt{2}\right)$; d) $a^2\left(12\sqrt{6}+5\sqrt{3}\right)$. **11.** a) 55; b) -1; c) 54; d) 56; e) 137. **12.** a) $A = 6a^2 + 2b^2$; $a = 2$; $b = -4 \Rightarrow A = 56$; b) $a^2 \geq 0\ \forall\ a \in \mathbb{R}$; $b^2 \geq 0\ \forall\ b \in \mathbb{R} \Rightarrow A \geq 0,\ \forall\ a, b \in \mathbb{R}$.

1.2. Înmulţirea, împărţirea şi ridicarea la putere a numerelor reale reprezentate prin litere

1. a) $-26x$; b) $15x^2y$; c) $-66x^2ab$; d) $\frac{84}{5}a^2b^2x$; e) $-\frac{9}{32}ab$. **2.** a) $-9x$; b) 7; c) $\frac{2}{15}x$; d) $\frac{2}{3}a$; e) $-\frac{625}{972}$. **3.** a) $-\frac{1}{5}$; b) -2; c) $\frac{3}{4}$; d) $\frac{3}{4}$; e) $\frac{1}{a^2}$; f) $\frac{1}{33b}$; g) $\frac{2}{3a^2b}$. **4.** a) 3^5; b) $\left(\frac{2}{3}\right)^4$; c) $\left(\frac{1}{3}\right)^{15}$; d) 2^{21}; e) 3^2; f) a^6; g) $\left(\overline{xy}\right)^3$; h) $(ab)^3$; i) x^{2014}. **5.** a) x^9; b) b^{12}; c) x^9; d) y^{19}; e) 1; f) b^3; g) x; h) 1. **6.** a) x^{42}; b) b^{42}; c) a^{40}; d) $81x^{20}y^{16}$; e) $81x^{20}y^{16}$; f) $-8x^{15}z^6$; g) $\frac{x^{36}y^{48}}{a^{60}}$; h) $\frac{64a^{12}b^{18}}{5^6x^6}$. **7.** a) $-16a^2b^3$; b) $12x^2y^2$; c) $-6a^2b$; d) $-6x^3y$. **8.** a) $5x + 4xy - 7x^2$; b) $-a^2 - 2b$; c) $\frac{11y}{2}-\frac{8x}{3}$; d) $2b - 16a$. **9.** a) $2a^2 + 5a - 3$; b) $-2a^2 + a\sqrt{2} + 2$; c) $-a^2 + 6a - 9$; d) $2a^2 - a - 15$; e) $21a^2 - 64a + 35$. **10.** a) $2a^4 - 17a^3 + 50a^2 - 61a + 26$; b) $-a^3 - 6a + 7$; c) $3a^2 - 5ac + ab - 3bc + c^2 - 5b^2$; d) $-2a^3 - 14a^2 - a - 90$. **11.** a) $2a + 3$; b) $a^2 + 2a + 1$; c) 1; d) $3a + 1$. **12.** a) $3x^2$; b) $14y^2$; c) $27x^2$; d) $3\sqrt{5}y^3+5\sqrt{3}x^3$.

1.3. Formule de calcul prescurtat

1. a) $y^2 + 25 + 10y$; b) $4x^2 + 1 + 4x$; c) $4a^2 + 25b^2 + 20ab$; d) $x^2+\frac{1}{4}+x$; e) $9x^2+\frac{1}{16}+\frac{3}{2}x$; f) $30 + 10\sqrt{5}$; g) $7b^2+4\sqrt{3}b^2$; h) $13+4\sqrt{3}$; i) $30+12\sqrt{6}$. **2.** a) $16 + x^2 - 8x$; b) $25 + 4c^2 - - 2ac$; c) $25x^2 + 9y^2 - 30xy$; d) $y^2+\frac{1}{9}-\frac{2y}{3}$; e) $4a^2+\frac{1}{36}-\frac{a}{6}$; f) $7-4\sqrt{3}$; g) $13-2\sqrt{42}$; h) $12-4\sqrt{5}$; i) 8. **3.** a) $a^2 - 9$; b) $16 - x^2$; c) $16x^2 - 9y^2$; d) $\frac{a^2-36}{4}$; e) $\frac{4x^2-9}{36}$; f) 1; g) 5; h) 3. **4.** a) $a^2 - 2a + 14$; b) $6x^2 + 4y^2 - 2xy$; c) $16a^2 + 9b^2 - 14ab - 15b$; d) $\frac{19a^2+7a-18}{2}$. **5.** a) $62^2 = = (60 + 2)^2 = 3600 + 4 + 240 = 3844$; b) $71^2 = (70 + 1)^2 = 4900 + 1 + 140 = 5041$; c) $59^2 = = (60 - 1)^2 = 3600 + 1 - 120 = 3481$; d) $(8,9)^2 = \left(9-\frac{1}{10}\right)^2 = 81+\frac{1}{100}-\frac{9}{5}=\frac{7921}{100}=79,21$; e) $(5,1)^2=\left(5+\frac{1}{10}\right)^2=25+\frac{1}{100}+2\cdot5\cdot\frac{1}{10}=\frac{2501}{100}=25,01$; f) $(3,99)^2=\left(4-\frac{1}{100}\right)^2$; h) $199^2 = = (200 - 1)^2$; i) $(2,001)^2 = \left(2+\frac{1}{1000}\right)^2$. **6.** a) $31 \cdot 29 = (30 + 1)(30 - 1) = 900 - 1 = 899$; b) $19 \cdot 21 = (20 - 1)(20 + 1)$; c) $26 \cdot 34 = (30 - 4)(30 + 4)$; d) $999 \cdot 1001 = (1000 - 1)(1000 + 1)$; e) $149 \cdot 151 = (150 - 1)(150 + 1)$; g) $(40 - 4)(40 + 4)$; h) $(50 - 2)(50 + 2)$; i) $(30 - 5)(30 + 5)$. **7.** a) $-4a^2 -4a - 84$; b) $b^2 - 9b - 11$; c) $-15a^2 + 8a - 27$, d) $5b^2 - 24b + 6$. **8.** $-\frac{3}{4}x^2-9x-4$; b) $\frac{78x^2}{9}-7x-9$; c) $\frac{1121x^2}{36}+\frac{17x}{6}-\frac{43}{144}$. **9.** a) $-8x^2 + 13 + 2\sqrt{2}x$; b) $26x^2 + 11$; c) $7x^2 +6x - - 13$; d) $14x^2 - 7$. **10.** a) $16a^2 - 39$; b) $-2b^2 - 31$; c) $48 - 3\sqrt{3}$; d) -46; **11.** a) 23; b) 110; c) 527. **12.** a) 18; b) 76; c) 322.

1.4. Metode de descompunere în factori

1. a) $2014(950 - 949) = 2014$; e) $x(a - x^2 + 2)$; f) $2(2x + 5y - 7z)$. **2.** a) $(y + 1)(a - 2 + 5a) = = (y + 1)(a - 2 + 5a) = (y + 1)(6a - 2) = 2(y + 1)(3a - 1)$. **3.** a) $(x + y)^2$; b) $(x - 2y)^2$; c) $(x - 10) \cdot \cdot (x + 10)$; d) $\left(x+\sqrt{6}\right)^2$; e) $\left(x-\sqrt{5}\right)^2$; f) $\left(2x-\sqrt{5}\right)\left(2x+\sqrt{5}\right)$. **4.** a) $\left(x-\sqrt{7}\right)\left(x+\sqrt{7}\right)$; b) $\left(2a-\frac{3}{5}\right)^2$; c) $\left(\frac{2x}{3}-\frac{1}{5}\right)\left(\frac{2x}{3}+\frac{1}{5}\right)$; d) $(x + 2yz)^2$; e) $(13x - 1)^2$; f) $(11x + 5a)^2$. **5.** a) $(a + b) \cdot \cdot (3 + x^2)$; b) $(a - 1)(3a^2 + 5)$; c) $(x + 1)(x - 2)(x + 2)$; d) $(a + 1)(x - 3)(x + 3)$; e) $(x + 1)(x - 3) \cdot \cdot (x^2 + 3x + 9)$; f) $(11x - 5a)^2$. **6.** a) $(x - 4)(x + 2)$; b) $a(a + 4)$; c) $(2b + 1)(2b + 5)$; d) $(x - 1 - y) \cdot \cdot (x - 1 + y)$; e) $(3x - 1)(7x + 1)$; f) $4(3x + 2)(x - 1)$. **7.** a) $25x^2y^2z^2(2yz^2 - 3x - 4x^2y + 1)$; b) $\frac{1}{3}x^3\left(x^3-\frac{2}{3}x-\frac{5}{4}x^2+\frac{1}{2}\right)$; e) $x^4\sqrt{5}\left(5x^2-4x+3x^3+6\right)$; f) $25x^m(1 - 3x + 2x^2 - 5x^3)$. **8.** a) $8(3x + 5)(x + 2)$; b) $(2x + 3)(4 - x)$; e) $(x - 4)(-x + 8)$; f) $(3x + 2)^2(3x + 1)$. **9.** a) $(x + 3)(x - 1)$;

b) $(5x+9)(x+9)$; e) $(16x^2+y^2)(4x-y)(4x+y)$; f) $\left(x+3\sqrt{2}\right)\left(x-\sqrt{2}\right)$. **11.** d) $(x^2-y^2)-(x+3)^2 =$ $= (x^2-y-x-3)(x^2-y+x+3)$; e) $\left[\left(x+\sqrt{2}\right)^2+\left(\sqrt{2}\right)^2\right]\left[\left(x+\sqrt{2}\right)^2-\left(\sqrt{2}\right)^2\right]=$ $=\left(x^2+2x\sqrt{2}+4\right)\left(x+2\sqrt{2}\right)\cdot x$; f) $\left[\left(2x+\sqrt{5}\right)^2+\left(x-\sqrt{5}\right)^2\right]\left[\left(2x+\sqrt{5}\right)^2-\left(x-\sqrt{5}\right)^2\right]=$ $=\left(5x^2+2x\sqrt{5}+10\right)\cdot 3x\cdot\left(x+2\sqrt{5}\right)$. **12.** a) $(x+2)(x+3)$; $(x+3)(x+4)$; $(x+2)(x+5)$; $(x+4)\cdot$ $\cdot(x+5)$; b) $(x-3)(x-2)$; $(x-3)(x-4)$; $(x-2)(x-5)$; $(x-4)(x-5)$; c) $(x+3)(x+5)$; $(x+2)(x+7)$; $(x+5)(x+6)$; $(x+7)(x+8)$; d) $(x+1)(x-11)$; $(x+1)(x-13)$; $(x-1)(x+14)$; $(x-1)(x+7)$; e) $(x+2)(x-3)$; $(x-2)(x+5)$; $(x-2)(x+6)$; $(x-2)(x+7)$; f) $(x-1)(x+3)$; $(x-1)(x+2)$; $(x-2)(x+3)$; $(x-2)(x+4)$.

1.5. Probleme pentru olimpiade şi concursuri

1. Avem $(x-y)^2 = 6xy$, $(x+y)^2 = 10xy$, de unde $\dfrac{x-y}{x+y}=\dfrac{\sqrt{6xy}}{\sqrt{10xy}}=\sqrt{\dfrac{6}{10}}=\sqrt{\dfrac{15}{5}}$; $\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}=$ $=\dfrac{\left(\sqrt{x}-\sqrt{y}\right)^2}{x-y}=\dfrac{x+y-2\sqrt{xy}}{x-y}=\dfrac{\sqrt{10}\sqrt{xy}-2\sqrt{xy}}{\sqrt{6}\cdot\sqrt{xy}}=\dfrac{\sqrt{10}-2}{\sqrt{6}}=\dfrac{2\sqrt{15}-\sqrt{6}}{3}$. **2.** Avem $ax = by =$ $= cz = dt = A+1$. Obţinem prin înmulţire $AB = (A+1)^4$, iar prin adunare $1 = 4A+4$. Avem $A =$ $=-\dfrac{3}{4}$; $B=-\dfrac{1}{192}$. **3.** $\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=\dfrac{1}{x+y+z}\Leftrightarrow (x+y+z)(xy+yz+zx)=xyz \Leftrightarrow (x+y)(xy+$ $=yz+zx)+z(xy+yz+zx)=xyz \Leftrightarrow (x+y)(xy+yz+zx)+z^2(x+y)=0 \Leftrightarrow (x+y)(y+z)(z+x)=$ $=0 \Leftrightarrow x+y=0$ sau $y+z=0$ sau $z+x=0$. Fie $x+y=0$. Atunci, pentru $n=2m+1$, $m\in\mathbb{N}$, avem $\dfrac{1}{x^n}+\dfrac{1}{y^n}+\dfrac{1}{z^n}=\dfrac{1}{z^n}=\dfrac{1}{x^n+y^n+z^n}$. **4.** $\dfrac{a+b}{2}+\sqrt{ab}=9\Leftrightarrow\left(\sqrt{a}+\sqrt{b}\right)^2=18\Leftrightarrow\sqrt{a}+\sqrt{b}=3\sqrt{2}$. Se arată că $a=2n^2$, $b=2m^2$ şi deci $n+m=3$. Avem perechile (9, 0), (8, 2), (2, 8), (0, 9). **5.** $A=a^2c^2+a^2d^2+b^2c^2+b^2d^2=(ac+bd)^2+(ad-bc)^2$ sau $A=(ac-bd)^2+(ad+bc)^2$. **6.** Deoarece avem $1^3+2^3+3^3+\ldots+n^3=\left(\dfrac{n(n+1)}{2}\right)^2$, rezultă că orice număr de forma $\left(\dfrac{n(n+1)}{2}\right)^2$ se scrie sub forma cerută cum $441=21^2=(1+2+3+4+5+6)$ avem $441=1^3+$ $+2^3+3^3+4^3+5^3+6^3$. **7.** $8p+1=(2n+1)^2\Rightarrow 2p=n(n+1)\Rightarrow P=3\Rightarrow 8p+1=5^2$. **8.** $(ac+$ $+bd)^2+(ad-bd)^2=4(ac+bd)^2\Rightarrow(ad-bc)^2=3(ac+bd)^2\Rightarrow ad=bc$, $ac+bd=0$ etc. **9.** Fie $x=\underbrace{11...1}_{n}\geq 11$. Avem $\overline{\underbrace{aa...a}_{2n}}=a\cdot\left(\underbrace{11...1}_{n}\underbrace{00...0}_{n}+\underbrace{11...1}_{n}\right)=a(x\cdot 10^n+x)=ax(10^n+1)=ax(9x+2)=$ $=9ax^2+2ax$. Obţinem $9ax^2+2ax-bx=c^2x^2$, de unde $x(c^2-9a)=2a-b$. Cazul 1: $c^2=9a$, $2a=b\Rightarrow a=1$, $b=2$, $c=3$, $n\geq 2$; Cazul 2: $c^2\neq 9a\Rightarrow x=\dfrac{2a-b}{c^2-9a}=11\Rightarrow a=7$, $b=3$, $c=8$, $n=2$. **10.** Fie $n+13=a^3$, $n-13=b^3$, $a, b\in\mathbb{N}^*$. Din $a^3-b^3=(a-b)(a^2+ab+b^2)=26$ rezultă că $a-b\leq 2$. Pentru $a=b+2$ rezultă $a=3$, $b=1$, $n=14$; Pentru $a=b+1$ nu avem soluţie.

11. Pentru $n = 0$ avem $36^n + 6^n + 3 = 5 = 2 + 3$. Fie $n \in \mathbb{N}^*$. Atunci $36^n + 6^n + 3 = 2 + p$, unde p este număr prim. Rezultă, pentru $6^n = x$, ecuația $x(x + 1) = p - 1 = 2m$, $m \in \mathbb{N}$, $m \geq 22$. Atunci $x = 2$, $p = 5$ (fals). **12.** Avem $(x^3 + y^3)^2 = (x + y)(x^5 + y^5) \Leftrightarrow xy(x - y)^2(x + y)^2 = 0$. Luăm cazurile $x = 0$, $y = 0$; $x = y$, $x = -y$. **13.** Avem $a - b = b^2 - a^2 \Leftrightarrow (a - b)(a + b + 1) = 0$ şi analog $(b - c)(b + c + 1) = 0$, $(a - \mathrm{c})(a + c + 1) = 0$. Dacă $a = b = c$ rezultă $2a^2 = a$ şi deci $a \in \left\{0, \dfrac{1}{2}\right\}$; Dacă $a = b \neq c$ nu avem soluție; Dacă $a + b + 1 = a + c + 1 = b + c + 1$, din nou nu avem soluție. **14.** $x^2 + y^2 + z^2 = 5^{2n-2} \cdot (25 - 5 - 1) = 5^{2n-2} \cdot (3^2 + 3^2 + 1)$. Fie $a = 5^{n-1}$. Luăm $(x, y, z) \in \{(3a, 3a, a), (3a, a, 3a), (a, 3a, 3a)\}$. **15.** Fie $\sqrt{x} = a > 0$ şi $\sqrt{32 - x} = \mathrm{b} > 0$. Avem $32 = a^2 + b^2 \geq 2ab$, de unde $ab \leq 16$, $\sqrt{ab} \leq 4$. Avem $A = \left(1 + \dfrac{1}{a}\right)\left(1 + \dfrac{1}{b}\right) = 1 + \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{ab} \geq 1 + \dfrac{1}{ab} + \dfrac{2}{\sqrt{ab}} \geq$ $\geq 1 + \dfrac{1}{16} + \dfrac{2}{4} = \dfrac{25}{16}$. **16.** Avem $A = (a - 2)^3 + (a - 1)^3 + a^3 + (a + 1)^3 + (a + 2)^3 = 5a^3 + 30a$. Luăm $a = 5^n$. **17.** Fie $s = a + b + c$. Din $100 \leq s^3 \leq 999 \Rightarrow s \in \{4, 5, 6, 7, 8, 9\}$. Avem $s^3 = 100a + 10b + c = 9(11a + b) + s$, de unde $(s - 1)s(s + 1) = 9(11a + b)$. Cum $s^3 - s \in \{3 \cdot 4 \cdot 5; 4 \cdot 5 \cdot 6; 5 \cdot 6 \cdot 7; 6 \cdot 7 \cdot 8; 7 \cdot 8 \cdot 9; 8 \cdot 9 \cdot 10\}$ şi $9 / s^3 - s$ rezultă că rămân cazurile $7 \cdot 8 \cdot 9$ şi $8 \cdot 9 \cdot 10$, de unde $s \in \{8, 9\}$. Dacă $s = 8$ avem $11a + b = 56$ şi atunci $\overline{abc} = 512$. Pentru $s = 9$ nu avem soluție. **18.** Deoarece $x + y + z = 1$ şi $x^3 + y^3 + z^3 = 1$, rezultă că $x^3 + y^3 + z^3 = (x + y + z)^3 = x^3 + y^3 + z^3 + 3(x + y)(\mathrm{y} + \mathrm{z})(z + x)$, de unde $x + y = 0$; sin $y + z = 0$ sin $z + x = 0$. Dacă (de exemplu) $z + x = 0$, avem $x^{2n+1} + y^{2n+1} + z^{2n+1} = x^{2n+1} + 1^{2n+1} + (-x)^{2n+1} = 1$. **19.** Avem $2^{2n} = 2^{n+2} \pm 2^{n+1} + 1 = 2^{2n} \pm 2^{n+1}(2 - 1) + 1 = 2^{2n} \pm 2 \cdot 2^n + 1 = (2^n \pm 1)^n$, de unde $2^n + 1 + 2^n - 1 = 2048 \Leftrightarrow$ $\Leftrightarrow 2^{n+1} = 2^{11} \Leftrightarrow n = 10$. **20.** $A = \dfrac{a}{ab + a + 1} + \dfrac{^{a)}b}{bc + b + 1} + \dfrac{^{ab)}c}{ca + c + 1} = \dfrac{a}{ab + b + 1} + \dfrac{ab}{1 + ab + a} + \dfrac{1}{a + 1 + ab} = 1$.

TESTE DE EVALUARE

Testul 1

I. 1. -6. **2.** $E(0) = 0$; $E(-1) = 0$. **3.** $a + b = 6$. **4.** $(x + 3 + y)(x + 3 - y)$. **5.** $\dfrac{x^3 - x^2}{x^2 - 1}$ şi $\dfrac{x}{x^2 - 1}$.

II. 1. a) $E(x) = (x^4 + 2x^2 + 1) - 2x(x^2 + 1) = (x^2 + 1)^2 - 2x(x^2 + 1) = (x^2 + 1)(x^2 + 1 - 2x) = (x^2 + 1)(x - 1)^2$; b) $E(x) = (x^2 + 1)(x - 1)^2 \geq 0$, $\forall\, x \in \mathbb{R}$, pentru că $x^2 + 1 > 0$ şi $(x - 1)^2 \geq 0$.

2. a) $a^2 + b^2 - 2ab = (a - b)^2 \geq 0$, $\forall\, a, b \in \mathbb{R} \Rightarrow a^2 + b^2 \Rightarrow a^2 + b^2 \geq 2ab$; b) $2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = (a - b)^2 + (a - c)^2 + (b - c)^2 \geq 0$, $\forall\, a, b, c \in \mathbb{R} \Leftrightarrow 2(a^2 + b^2 + c^2) \geq 2(ab + bc + ca) \Leftrightarrow a^2 + b^2 + c^2 \geq ab + bc + ac$.

III. 1. a) $E(x) = x^2 + 7x - 5x - 35 = x(x + 7) - 5(x + 7) = (x - 5)(x + 7)$; $E(x) = 0 \Leftrightarrow x_1 = 5$ şi $x_2 = -7$; b) $E(n) = (n - 5)(n + 7)$ prim $\Rightarrow n - 5 = 1 \Rightarrow n = 6 \Rightarrow E(6) = 13$ sau $n + 7 = 1 \Rightarrow n = -6 \Rightarrow E(-6) = -11 \notin \mathbb{N}$; c) $E(x) = (x - 5)(x + 7) = \mathcal{M}_3 \Rightarrow x - 5 = \mathcal{M}_3 \Rightarrow x + 7 = \mathcal{M}_3 + 12 = \mathcal{M}_3 \Rightarrow E(x) = \mathcal{M}_9$ sau $x + 7 = \mathcal{M}_3 \Rightarrow x - 5 = \mathcal{M}_3 - 12 = \mathcal{M}_3 \Rightarrow E(x) = \mathcal{M}_9$.

Testul 2

I. 1. x^2. **2.** $E(2) = 0$. **3.** $a^2 + b^2 = 12$. **4.** $(3x - 2)(-x - 2)$. **5.** $\dfrac{x+1}{x+5}$.

II. 1. a) $E(x) = x^2(x + 4) - (x + 4) = (x + 4)(x^2 - 1) = (x + 4)(x - 1)(x + 1)$;
b) $E(x) \cdot F(x) = (x^3 + 4x^2 - x - 4)(ax^2 + bx - 3) = \ldots$;
$x^4 : b + 4a = 6$; $x^3 : -3 + 4b - a = 4 \Rightarrow$
$$\begin{aligned} &b + 4a = 6 \\ &\underline{4b - a = 7\ \ } \mid \cdot 4 \\ &17b = 34 \Rightarrow b = 2 \Rightarrow a = 1 \end{aligned}$$

2. a) $E(x) = x^2 + 4x + 5 = x^2 + 4x + 4 + 1$ (1)

$E(x) = (x + a)^2 + 1, \forall x \in \mathbb{R}$ (2)

Din (1) şi (2) $\Rightarrow a = 2$.

b) $E(x) \geq 1$ pentru că $E(x) = (x + 2)^2 + 1$ şi $(x + 2)^2 \geq 0$, $\forall x \in \mathbb{R}$. $E(x)$ minimă $\Rightarrow E(x) = 1 \Leftrightarrow$

$\Leftrightarrow (x + 2)^2 = 0 \Leftrightarrow x = -2$.

III. 1. a) $x \cdot y = \sqrt{\left(4-\sqrt{7}\right)\left(4+\sqrt{7}\right)} = \sqrt{16-7} = 3$; b) $(x - y)^2 = x^2 + y^2 - 2xy = 4 - \cancel{\sqrt{7}} +$

$+ 4 + \cancel{\sqrt{7}} - 6 = 2$; c) conform b), $(x - y)^2 = 2 \Rightarrow \left.\begin{array}{l} |x-y| = \sqrt{2} \\ \text{evident } x < y \end{array}\right\} \Rightarrow x - y = -\sqrt{2} \Rightarrow \dfrac{x-y}{\sqrt{2}} =$

$= -1 \in \mathbb{Z}_-$.

Testul 3

I. 1. $2x^2 + 2x + 1$. **2.** 35. **3.** 2. **4.** $(x + 1)(x - 4)(x + 4)$. **5.** $\dfrac{x+2}{x-3}$.

II. 1. a) $E(n) = n^2 + 2n - 35 = n^2 + 7n - 5n - 35 = n(n + 7) - 5(n + 7) = (n - 5)(n + 7)$ prim $\Rightarrow$
$\Rightarrow n - 5 = 1 \Rightarrow n = 6 \Rightarrow E(6) = 13$ sau $n + 7 = 1 \Rightarrow n = -6 \Rightarrow E(-6) = -11 \notin \mathbb{N} \Rightarrow n = 6$;

b) $E(n) = (n - 5)(n + 7)$, $3 \mid E(n) \Rightarrow 3 \mid n - 5 \Rightarrow n + 7 = \mathcal{M}_3 + 12 = \mathcal{M}_3 \Rightarrow$

$\Rightarrow \left.\begin{array}{l} 3 \mid n-5 \\ 3 \mid n+7 \end{array}\right\} \Rightarrow 9 \mid E(n)$ sau $3 \mid n + 7 \Rightarrow n - 5 = \mathcal{M}_3 - 12 = \mathcal{M}_3 \Rightarrow E(n) = \mathcal{M}_9$.

2. $(x^2 - 4x + 4) + (3y^2 + 2\sqrt{3}\,y + 1) = (x - 2)^2 + \left(\sqrt{3}y + 1\right)^2 \geq 0$, $\forall x, y \in \mathbb{R}$.

III. 1. a) $p^2 = 8 - \cancel{2\sqrt{15}} - 2\sqrt{\left(8-2\sqrt{15}\right)\left(8+2\sqrt{15}\right)} + 8 + \cancel{2\sqrt{15}} = 16 - 2\sqrt{64-60} = 16 - 4 =$

$= 12$; b) conform a) $\Rightarrow p^2 = 12 \Rightarrow p = \pm 2\sqrt{3}$, dar $p < 0 \Rightarrow p = -2\sqrt{3} \Rightarrow \left(p + 2\sqrt{3}\right)^{2001} = 0$.

Testul 4

I. 1. $2x^2 + 2x + 2$. **2.** $5 + \sqrt{3}$. **3.** 6. **4.** $(x + 1)(x + 6)$. **5.** $x_1 = 1$, $x_2 = \dfrac{2}{3}$, $x_3 = -\dfrac{2}{3}$.

II. 1. a) $2x^2 - 5xy + 3y^2 = 0 \mid : y^2 \neq 0 \Leftrightarrow 2\left(\dfrac{x}{y}\right)^2 - 5\dfrac{x}{y} + 3 = 0$; $\dfrac{x}{y} = t$, $2t^2 - 5t + 3 = 0 \Leftrightarrow$

$\Leftrightarrow 2t^2 - 2t - 3t + 3 = 0 \Leftrightarrow 2t(t - 1) - 3(t - 1)$; $t_1 = 1$ nu convine pentru că $x \neq y$; $t_2 = \dfrac{3}{2} \Rightarrow \dfrac{x}{y} = \dfrac{3}{2}$.

b) $(x^2 + 6x + 9) + (4y^2 - 4y + 1) = 0 \Leftrightarrow (x + 3)^2 + (2y - 1)^2 = 0 \Leftrightarrow x = -3$ şi $y = \dfrac{1}{2}$.

2. $a = \left(\dfrac{1}{2} - \sqrt{3}\right)^2 + \left(\dfrac{\sqrt{3}}{2} + 1\right)^2 + \left(1 - \sqrt{5}\right)\left(1 + \sqrt{5}\right) = \dfrac{1}{4} - \cancel{\sqrt{3}} + 3 + \dfrac{3}{4} + \cancel{\sqrt{3}} + 1 + 1 - 5 = 1 \in \mathbb{Z}.$

III. 1. a) $\sqrt{\left(1 - \sqrt{2}\right)^2} - \sqrt{2} = \left|1 - \sqrt{2}\right| - \sqrt{2} = \sqrt{2} - 1 - \sqrt{2} = -1$; b) $9n^2 + 6n + 1 = (3n + 1)^2$, $\forall\, n \in \mathbb{N}$; c) $E(x) = \sqrt{(x-3)^2} + \sqrt{(3y+1)^2 + 9} \geq \sqrt{9} = 3$ pentru $x = 3$ şi $y = -\dfrac{1}{3}$;

$E_{\min} = E\left(3, -\dfrac{1}{3}\right) = 3$.

CAPITOLUL 2
Ecuaţii şi inecuaţii

2.1. Proprietăţi ale relaţiei de egalitate în mulţimea numerelor reale

1. $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$. **2.** $\left[\left(\dfrac{1}{2} + \dfrac{1}{3}\right) : \dfrac{5}{6}\right]^{2015} = \left(\dfrac{5}{6} : \dfrac{5}{6}\right)^{2015} = 1$. **3.** $\left(2 + \sqrt{2}\right)\left(2 - \sqrt{2}\right) = 4 - 2 = 2$.

4. $1 + 2\sqrt{3} + 3 + 1 - 2\sqrt{3} + 3 = 8$. **5.** $2 - \sqrt{3} + \sqrt{3} + 2 = 4$. **6.** $\dfrac{2\left(\sqrt{5} + \sqrt{3}\right)}{2} - \dfrac{2\left(\sqrt{3} + 1\right)}{2} - \sqrt{5} = -1$.

7. a) $\left|2 - \sqrt{3}\right| + \left|2 + \sqrt{3}\right| = 4$; b) $\left|5 - 2\sqrt{6}\right| + \left|5 + 2\sqrt{6}\right| - 8 = 5 - 2\sqrt{6} + 5 + 2\sqrt{6} - 8 = 2$;

c) $\left(3^{222} - 3^{33} + 2^{333}\right) : 3^{222} = 1$. **8.** a) $\left(1 + \sqrt{2} + 1 - \sqrt{2}\right)^2 = 4$; b) $\left(3 + \sqrt{5} - 3 + \sqrt{5}\right)^2 = 20$;

c) $4 + 2\sqrt{3} - 2\sqrt{16 - 12} + 4 - 2\sqrt{3} = 4$. **9.** a) $a^2 + 2a + 1 + a^2 - 2a + 1 - 2a^2 = 2$; b) $2a^2 + 2a\sqrt{6} + 3 + 2a^2 - 2a\sqrt{6} + 3 - 4a^2 = 6$; c) $(a - 1)(a + 1)(a^2 + 1)(a^4 + 1)(a^8 + 1) = (a^2 - 1)(a^2 + 1)(a^4 + 1)(a^8 + 1) = (a^4 - 1)(a^4 + 1)(a^8 + 1) = (a^8 - 1)(a^8 + 1) = a^{16} - 1$. **10.** a) $(2x + 3)[(x + 1)^2 - 4(x + 1) + 4] = (2x + 3)(x + 1 - 2)^2$; b) $(3x + 3)[(3x + 1)^2 + 4(3x + 1) + 4] = (3x + 3)(3x + 3)^2 = (3x + 3)^3$; c) $(2x + 5)^2[(2x + 1)^2 + 8(2x + 1) + 16] = (2x + 5)^2(2x + 1 + 4)^2 = (2x + 5)^4$. **11.** $2x + 3y = 4$; $(2x - 3y)(2x + 3y) = 64 \Rightarrow 2x - 3y = 16 \Rightarrow x = 5, y = -2$. **12.** a) $2x + y = 5 \Rightarrow (2x + y)^2 = 25 \Rightarrow 4x^2 + 4xy + y^2 = 25 \Rightarrow xy = 2$; b) $(2x - y)^2 = 4x^2 - 4xy + y^2 = 17 - 8 = 9$; c) $2x + y = 5$; $2x - y = 3 \Rightarrow x = 2$; $y = 1$; $2x + y = 5$; $2x - y = -3 \Rightarrow x = \dfrac{1}{2}$; $y = 4$. **13.** $1 + 2 + 2^2 + 2^3 + \ldots + 2^{2015} = S$; $2S = 2 + 2^2 + 2^3 + 2^4 + \ldots + 2^{2016} \Rightarrow 2S - S = 2^{2016} - 1 \Rightarrow S = 2^{2016} - 1$. **14.** $4ab = (a + b)^2 \Rightarrow (a - b)^2 = 0 \Rightarrow a = b$. **15.** $(a + b + c)^2 = 3(a^2 + b^2 + c^2) \Leftrightarrow a^2 + 2ab + b^2 + c^2 + 2ac + 2bc = 3a^2 + 3b^2 + 3c^2 \Leftrightarrow 2a^2 - 2ab + 2b^2 - 2ac + 2c^2 - 2bc = 0 \Leftrightarrow a^2 - 2ab + b^2 + a^2 - 2ac + c^2 + b^2 - 2bc + c^2 = 0 \Leftrightarrow (a - b)^2 + (a - c)^2 + (b - c)^2 = 0 \Leftrightarrow a = b = c$. **16.** $\dfrac{a}{b} + \dfrac{b}{a} + \dfrac{a}{c} + \dfrac{c}{a} + \dfrac{b}{c} + \dfrac{c}{b} - 6 = 0 \Leftrightarrow \dfrac{(a-b)^2}{ab} + \dfrac{(a-c)^2}{ac} + \dfrac{(b-c)^2}{bc} = 0 \Rightarrow a = b = c$.

2.2. Ecuaţii de gradul I cu o necunoscută

1. a) $3 \cdot 1 + 1 = 2(1 + 3) - 4 \Leftrightarrow 3 + 1 = 8 - 4 \Leftrightarrow 4 = 4 \Rightarrow 1$ este soluție a ecuației; b) $3(-2 -2) + 2(-2 + 5) = 4(-2 -1) + 10 \Leftrightarrow -12 + 6 = -12 + 10 \Leftrightarrow -6 = -2 \Rightarrow -2$ nu este soluție a ecuației; c) $|-4 + 1| - 4 = -1 \Leftrightarrow 3 - 4 = -1 \Leftrightarrow -1 = -1 \Rightarrow -4$ este soluție a ecuației; d) $\left(\sqrt{2}-1\right)\left(\sqrt{2}-1\right) - 3 + 2\sqrt{2} = 0 \Leftrightarrow 2-\sqrt{2}-\sqrt{2}+1-3+2\sqrt{2}=0 \Leftrightarrow 0=0 \Rightarrow \sqrt{2}-1$ este soluție a ecuației. **2.** a) $3 \cdot (-1) + m = 5(-1) + 2 \Leftrightarrow -3 + m = -5 + 2 \Leftrightarrow -3 + m = -3 \Leftrightarrow m = 0$; b) $3m + 4(-1) = 6 + 5 - 3m \Leftrightarrow 6m = 11 + 4 \Leftrightarrow 6m = 15 \Leftrightarrow m = \frac{15}{6} \Leftrightarrow m = \frac{5}{2}$; c) $\frac{2}{3}\sqrt{3}-\frac{m}{\sqrt{3}}=0 \Leftrightarrow m = \frac{2\sqrt{3}\cdot\sqrt{3}}{3} \Leftrightarrow m = 2$. **3.** a) $6x = 12 \Leftrightarrow x = 2$ este soluție în $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ şi $\mathbb{R}$; b) $3x + 7 = 0 \Rightarrow x = -\frac{7}{3}$ este soluție în $\mathbb{Q}$ şi $\mathbb{R}$, dar nu în $\mathbb{N}$ sau $\mathbb{Z}$; c) $3x = -6 \Leftrightarrow x = -2$ este soluție în $\mathbb{Z}$, $\mathbb{Q}$ şi $\mathbb{R}$ dar nu şi în $\mathbb{N}$; d) $2{,}7x = 5{,}4 \Leftrightarrow x = 2$ este soluție în $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ şi $\mathbb{R}$; e) $\frac{1}{2}x=-\frac{5}{4} \Leftrightarrow x=-\frac{5}{4}\cdot\frac{2}{1} \Leftrightarrow x=-\frac{5}{2}$ este soluție în $\mathbb{Q}$ şi $\mathbb{R}$, dar nu şi în $\mathbb{N}$ sau $\mathbb{Z}$; f) $\frac{1}{8}x=-\frac{3}{4} \Leftrightarrow x=-\frac{3}{4}\cdot\frac{8}{1} \Leftrightarrow x=-6$ este soluție în $\mathbb{Z}$, $\mathbb{Q}$ şi $\mathbb{R}$ dar nu şi în $\mathbb{N}$; g) $\sqrt{3}x = 2\cdot3\sqrt{3} \Leftrightarrow x=\frac{6\sqrt{3}}{\sqrt{3}} \Leftrightarrow x = 6$ este soluție în $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$; h) $\sqrt{5}x=-\sqrt{15} \Leftrightarrow x=-\sqrt{3}$ este soluție în $\mathbb{R}$, dar nu şi în $\mathbb{N}$, $\mathbb{Z}$ sau $\mathbb{Q}$. **4.** a) $9x + 5 - x = 4x + 4 \Leftrightarrow 8x - 4x = 4 - 5 \Leftrightarrow 4x = -1 \Rightarrow x=-\frac{1}{4}$; b) $6x - 15 + x + 4 = 3x - 1 \Leftrightarrow 7x- 3x = -1 + 15 - 4 \Leftrightarrow 4x = 10 \Leftrightarrow x=\frac{10}{4} \Leftrightarrow x=\frac{5}{2}$; c) $5 - 6x + 10 + 8x - 13 = 0 \Leftrightarrow 2x = 13 - 15 \Leftrightarrow 2x = -2 \Leftrightarrow x = -1$; d) $\sqrt{3}x-3\sqrt{3}+5\sqrt{3}=0 \Leftrightarrow \sqrt{3}x=-2\sqrt{3} \Leftrightarrow x = -2$; e) $\left(6-\sqrt{5}\right)x=62 \Leftrightarrow x = \frac{62}{6-\sqrt{5}} \Leftrightarrow x = \frac{62\left(6+\sqrt{5}\right)}{36-5} \Leftrightarrow x = \frac{62\left(6+\sqrt{5}\right)}{31} \Leftrightarrow x = 2\left(6+\sqrt{5}\right)$; f) $x\left(\sqrt{10}-\sqrt{2}\right) = \sqrt{15}-\sqrt{3} \Leftrightarrow x\sqrt{2}\left(\sqrt{5}-1\right)=\sqrt{3}\left(\sqrt{5}-1\right) \Leftrightarrow x=\frac{\sqrt{3}}{\sqrt{2}} \Leftrightarrow x=\frac{\sqrt{6}}{2}$. **5.** a) $3x - 6 - 4x + 4 = -7x - 14 \Leftrightarrow 6x = -12 \Leftrightarrow x = -2$; $7x - 14 - 2x + 8 = -26 -5x \Leftrightarrow 5x + 5x = -26 + 14 - 8 \Leftrightarrow 10x = -20 \Leftrightarrow x = -2$, deci cele două ecuații sunt echivalente; b) $7\sqrt{3}x-\sqrt{48}=\sqrt{27} \Leftrightarrow 7\sqrt{3}x-4\sqrt{3} = 3\sqrt{3} \Leftrightarrow 7\sqrt{3}x=7\sqrt{3} \Leftrightarrow x=1$; $x^2 - 1 + 3 = x^2 - x + 7x - 4 \Leftrightarrow -6x = -6 \Leftrightarrow x = 1$, deci cele două ecuații sunt echivalente; c) $3x - 6 - 5x - 15 = -8x + 40 - 1 \Leftrightarrow -2x + 8x = 39 + 21 \Leftrightarrow 6x = 60 \Leftrightarrow x = 10$; $3 - 3\sqrt{5}x+\sqrt{5}x-2\sqrt{5}=\sqrt{5}x+\sqrt{5}+3 \Leftrightarrow -3\sqrt{5}x=3\sqrt{5} \Leftrightarrow x=-1$, deci cele două ecuații nu sunt echivalente. **6.** a) $6x + 4x - 6 - 40x - 5 = 43 - 36x \Leftrightarrow 6x = 54 \Leftrightarrow x = 9$; b) $2x - 6 + 6x - 12 = 3 + 5x - 10 \Leftrightarrow 3x = 11 \Leftrightarrow x=\frac{11}{3}$; c) $6x + 9 = 15x + 30 - 12x - 42 \Leftrightarrow 3x = -21 \Leftrightarrow x = -7$; d) $12x + 9 - 10x = x + 32 \Leftrightarrow x = 23$; e) $10x + 30 + 30x - 150 = 15x - 30 + 6x + 24 \Leftrightarrow 19x = 114 \Leftrightarrow x = 6$; f) $8x + 28 - 9x + 21 = 24 \Leftrightarrow -x = -25 \Leftrightarrow x = 25$;

g) $-\sqrt{3}x-2\sqrt{3}-10=20-4\sqrt{3}x+4\sqrt{3} \Leftrightarrow 3\sqrt{3}x=30+6\sqrt{3} \Leftrightarrow x=\dfrac{6\left(5+\sqrt{3}\right)}{3\sqrt{3}} \Leftrightarrow x=\dfrac{6+10\sqrt{3}}{3}$;

h) $2\sqrt{2}x-8+\sqrt{2}+3\sqrt{2}=2x-2\sqrt{2}-2x \Leftrightarrow 2\sqrt{2}x=-6\sqrt{2}+8 \Leftrightarrow x=\dfrac{2\left(4-3\sqrt{2}\right)}{2\sqrt{2}} \Leftrightarrow x=2\sqrt{2}-3$.

7. a) $3x-6-12+2x=-18 \Leftrightarrow 5x=0 \Leftrightarrow x=0$; b) $3x+6-2+2x=4x+5 \Leftrightarrow 5x-4x=5-4 \Leftrightarrow x=1$; c) $10x-5-12x+15=-3x+11 \Leftrightarrow x=1$; d) $8x-12+5+6-12x=7x-12-12x+6 \Leftrightarrow -4x+5x=-6+1 \Leftrightarrow x=-5$. **8.** a) $3x-2+2x=18 \Leftrightarrow 5x=20 \Leftrightarrow x=4$; b) $x+1-6=x-3-2 \Leftrightarrow -5=-5 \Rightarrow x\in\mathbb{R}$; c) $x+4-3x+9=24-2x-10 \Leftrightarrow -2x+2x=14-13 \Leftrightarrow 0=1 \Rightarrow x\in\varnothing$; d) $16x-2x+2=5x-15-10 \Leftrightarrow 9x=-27 \Leftrightarrow x=-3$; e) $6x-8x+8+9x-18-8x+24=6x-30+2 \Leftrightarrow -7x=-42 \Leftrightarrow x=6$; f) $6x-15=14+8x-6+4x+25 \Leftrightarrow 6x-12x=8+15+25 \Leftrightarrow -6x=48 \Leftrightarrow x=-8$. **9.** a) $x^2+2x+1+x^2-7x+12=x^2-2x+1+x^2+7x+12+10 \Leftrightarrow -10x=10 \Leftrightarrow x=-1$; b) $9x^2+6x+1+4x^2-4x+1=4x^2+4x+1+9x^2-6x+1-4 \Leftrightarrow 4x=-4 \Leftrightarrow x=-1$; c) $x^2-x^2+4x-4=x^2-2x+1-x^2-4x-4 \Leftrightarrow 10x=1 \Leftrightarrow x=\dfrac{1}{10}$;

d) $9-x^2=x^2+4x+4-2x^2+8x+10 \Leftrightarrow -5=12x \Leftrightarrow x=-\dfrac{5}{12}$; e) $4x^2+8x+4+5x^2+5x-10=1-6x+9x^2+12 \Leftrightarrow 19x=19 \Leftrightarrow x=1$; f) $2x+2-x^2+10x-25=3x+6-x^2-2x+4 \Leftrightarrow 12x-x=6+23+4 \Leftrightarrow 11x=33 \Leftrightarrow x=3$. **10.** a) $6x-4\sqrt{5}-6x+3\sqrt{5}x=2\sqrt{5} \Leftrightarrow 3\sqrt{5}x=6\sqrt{5} \Leftrightarrow x=2$; b) $6\sqrt{2}x^2-x\sqrt{6}-6\sqrt{2}x^2+6x=2\sqrt{3}-\sqrt{2} \Leftrightarrow x\sqrt{6}\left(\sqrt{6}-1\right)=$

$=\sqrt{2}\left(\sqrt{6}-1\right) \Leftrightarrow x=\dfrac{1}{\sqrt{3}}=\dfrac{\sqrt{3}}{3}$; c) $2\sqrt{5}x-20+\sqrt{5}=-2x+2x-2\sqrt{5}-3\sqrt{5} \Leftrightarrow 2\sqrt{5}x=-6\sqrt{5}+$

$+20 \Leftrightarrow x=\dfrac{2\left(-3\sqrt{5}+10\right)}{2\sqrt{5}} \Leftrightarrow x=-3+2\sqrt{5}$; d) $4+2\sqrt{3}+x\sqrt{3}+x+\sqrt{3}+1+1-2\sqrt{3}x-4\sqrt{3}=$

$=3 \Leftrightarrow x\left(1-\sqrt{3}\right)=\sqrt{3}-3 \Leftrightarrow x=\dfrac{\sqrt{3}-3}{1-\sqrt{3}} \Leftrightarrow x=\sqrt{3}$; e) $2x+2\sqrt{3}-5\sqrt{3}=2\sqrt{3}x+\sqrt{3}-4 \Leftrightarrow$

$\Leftrightarrow x\left(2-2\sqrt{3}\right)=4\sqrt{3}-4 \Leftrightarrow x=\dfrac{4\left(\sqrt{3}-1\right)}{2\left(1-\sqrt{3}\right)} \Leftrightarrow x=-2$; f) $2x\sqrt{3}+2x+3=3\sqrt{3}+\sqrt{3}x-x \Leftrightarrow$

$\Leftrightarrow x\left(\sqrt{3}+3\right)=3\sqrt{3}-3 \Leftrightarrow x=\dfrac{3\left(\sqrt{3}-1\right)}{3+\sqrt{3}} \Leftrightarrow x=\dfrac{3\left(\sqrt{3}-1\right)}{\sqrt{3}\left(\sqrt{3}+1\right)} \Leftrightarrow x=2\sqrt{3}-3$. **11.** a) $12-20+2-6=7x-5 \Leftrightarrow 7x=21 \Leftrightarrow x=3$; $6m-7m-14=9-4 \Leftrightarrow -m=19 \Leftrightarrow m=-19$; b) $15x+9x-27=60-25x+60 \Leftrightarrow 49x=147 \Leftrightarrow x=3$; $\dfrac{3m+3}{2}-3=5-\dfrac{3+m}{3} \Leftrightarrow 9m+9-18=30-6-2m \Leftrightarrow 11m=33 \Leftrightarrow m=3$; c) $x\left(\sqrt{6}+\sqrt{5}\right)-x\left(\sqrt{6}-\sqrt{5}\right)=4\sqrt{5} \Leftrightarrow x\left(\sqrt{6}+\sqrt{5}-\sqrt{6}+\sqrt{5}\right)=$

$=4\sqrt{5} \Leftrightarrow x=2$; $\dfrac{10m}{\sqrt{2}}-6\sqrt{2}=4\sqrt{2} \Leftrightarrow \dfrac{10m}{\sqrt{2}}=10\sqrt{2} \Leftrightarrow m=\dfrac{20}{2} \Leftrightarrow m=10$.

12. a) $\frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{2}x+3\right)+2\right]+2=4 \Rightarrow \frac{1}{2}x+3=4 \Leftrightarrow x=2$; b) $3|x| - 12 + 7 - 3|x| = |x| - 7 \Leftrightarrow$ $\Leftrightarrow -5 = |x| - 7 \Leftrightarrow |x| = 2 \Leftrightarrow x \in \{-2; 2\}$; c) $x \neq 3$; $\frac{2}{3(x-3)}-\frac{5}{4(x-3)}-\frac{3}{5(x-3)}+\frac{1}{x-3}=\frac{1}{120} \Leftrightarrow$ $\Leftrightarrow 80-150-72+120=x-3 \Leftrightarrow x=-19$. **13.** a) $x \neq \frac{3}{2}$; $\frac{3+2x}{3-2x}=\frac{5-3x}{2x-3} \Leftrightarrow 3+2x=3x-5 \Leftrightarrow$ $\Leftrightarrow x = 8$; b) $x \in \mathbb{R} \setminus \left\{\pm\frac{1}{3}\right\}$; $(1+3x)^2 + 12 = (1-3x)^2 \Leftrightarrow 1 + 6x + 9x^2 + 12 = 1 - 6x + 9x^2 \Leftrightarrow$ $\Leftrightarrow x = -1$; c) $x \in \mathbb{R} \setminus \{\pm 3\}$; $5x - 15 = x(x-3) - (x+1)(x+3) \Leftrightarrow 5x - 15 = x^2 - 3x - x^2 - 3x -$ $- x - 3 \Leftrightarrow 12x = 12 \Leftrightarrow x = 1$; d) $x \in \mathbb{R} \setminus \{3\}$; $3x + 1 + (2x+1)(x-3) = 2(x^2 - 6x + 9) \Leftrightarrow 3x +$ $+ 1 + 2x^2 - 6x + x - 3 = 2x^2 - 12x + 18 \Leftrightarrow 10x = 20 \Leftrightarrow x = 2$. **14.** a) $x\left|2-\sqrt{5}\right|-2\left|\sqrt{5}-2\right|=$ $=\left|3-\sqrt{5}\right|-x \Leftrightarrow x\left(\sqrt{5}-2\right)+x=3-\sqrt{5}+2\left(\sqrt{5}-2\right) \Leftrightarrow x\left(\sqrt{5}-1\right)=\sqrt{5}-1 \Leftrightarrow x=1$;

b) $\sqrt{(x-4)^2}-(2x-8)=0 \Leftrightarrow |x-4|=2(x-4) \Rightarrow x=4$;

c) $x\left|3-\sqrt{7}\right|-4x=\left|2-\sqrt{7}\right|+5-\sqrt{7}-x\left|2-\sqrt{7}\right| \Leftrightarrow x\left(3-\sqrt{7}\right)-4x+x\left(\sqrt{7}-2\right)=\sqrt{7}-2+\ 5-$ $-\sqrt{7} \Leftrightarrow x\left(3-\sqrt{7}-4+\sqrt{7}-2\right)=3 \Leftrightarrow -3x=3 \Leftrightarrow x=-1$.

2.3. Proprietăţi ale relaţiei de inegalitate dintre numerele reale

1. $a \geq 4 \Rightarrow -2a \leq -8 \Rightarrow 3 - 2a \leq -5$. **2.** $a \leq 1 \Rightarrow -2a \geq -2 \ |+3 \Rightarrow -2a + 3 \geq 1$. **3.** $a \geq 3 \Rightarrow$ $\Rightarrow 3a \geq 9 \ |+1 \Rightarrow 3a + 1 \geq 10 \ | : 2 \Leftrightarrow \frac{3a+1}{2} \geq 5$. **4.** $-1 \leq a \leq 3 \ | + 5 \Rightarrow 4 \leq a + 5 \leq 8 \ | : 4 \Rightarrow 1 \leq$ $\leq \frac{a+5}{4} \leq 2$.

5. $-2 \leq b \leq 2 \ |\cdot(-1) \Leftrightarrow$
$$\begin{array}{l} -2 \leq -b \leq 2 \\ \underline{-1 \leq \ a \leq 4} \\ -3 \leq a - b \leq 6. \end{array}$$

6. $a^2 = 7 + 2\sqrt{10}$; $b^2 = 7 + 2\sqrt{12}$
$\sqrt{10} < \sqrt{12} \Rightarrow a^2 < b^2$, $a, b > 0 \Rightarrow$
$\Rightarrow a < b$.

7. $a = \sqrt{11}+\sqrt{10}$; $b=\sqrt{7}+\sqrt{6}$; $\sqrt{11}>\sqrt{7}$, $\sqrt{10}>\sqrt{6} \Rightarrow \sqrt{11}+\sqrt{10}>\sqrt{7}+\sqrt{6} \Rightarrow a > b$.

8. $-1 \leq a \leq 1$
$\underline{-2 \leq 2b \leq 2}$
$-3 \leq a + 2b \leq 3 \Rightarrow a + 2b + 3 \geq 0 \Rightarrow |a + 2b + 3| = a + 2b + 3$; $-1 \leq a \leq 1| \cdot 2 \Rightarrow$
$-2 \leq 2a \leq 2$
$\underline{-1 \leq \ b \leq 1}$
$-3 \leq 2a + b \leq 3 \Rightarrow 2a + b + 3 \geq 0$ şi $2a + b - 3 \leq 0 \Rightarrow |2a + b - 3| = 3 - 2a -$ b şi
$A = a + 2b + 3 + 3 - 2a - b = 6 - a + b = 6 - (a - b) = 6-\frac{1}{3}=\frac{17}{3}$.

9. $\sqrt{13}-\sqrt{5}>\sqrt{14}-\sqrt{6} \Leftrightarrow \sqrt{13}+\sqrt{6}>\sqrt{14}+\sqrt{5} \Leftrightarrow 13+2\sqrt{78}+6>14+2\sqrt{70}+5 \Leftrightarrow 19 +$ $+2\sqrt{78}>19+2\sqrt{70} \Leftrightarrow \sqrt{78}>\sqrt{70}$. Adevărat. **10.** a) $x^2 + 4x + 8 = x^2 + 4x + 4 + 4 = (x+2)^2 + 4$, $\forall\ x \in \mathbb{R}$; b) $(x+2)^2 \geq 0$, $\forall\ x \in \mathbb{R} \Rightarrow (x+2)^2 + 4 \geq 4$, $\forall\ x \in \mathbb{R} \Leftrightarrow x^2 + 4x + 8 \geq 4$, $\forall\ x \in \mathbb{R}$.

11. a) $-x^2 + 6x - 5 = -(x^2 - 6x + 9 - 4) = -[(x-3)^2 - 4] = 4 - (x-3)^2$, $\forall\ x \in \mathbb{R}$; b) $(x-3)^2 \geq 0$,

$\forall x \in \mathbb{R} \Leftrightarrow -(x-3)^2 \le 0$, $\forall x \in \mathbb{R} \Leftrightarrow 4-(x-3)^2 \le 4$, $\forall x \in \mathbb{R} \Leftrightarrow -x^2+6x-5 \le 0$, $\forall x \in \mathbb{R}$.

12. $N = |2a+3b+5| + |2a+3b-5| + |3a+2b+5| + |3a+2b-5|$; $-1 \le a \le 1 \Rightarrow -2 \le 2a \le 2$ (1); $-1 \le b \le 1 \Rightarrow -3 \le 3b \le 3$ (2). Din (1) + (2) $\Rightarrow -5 \le 2a+3b \le 5 \Leftrightarrow \begin{cases} 2a+3b+5 \ge 0 \\ 2a+3b-5 \le 0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} |2a+3b+5| = 2a+3b+5 \\ |2a+3b-5| = 5-2a-3b \end{cases}$. $-1 \le a \le 1 \Rightarrow -3 \le 3a \le 3$ (3); $-1 \le b \le 1 \Rightarrow -2 \le 2b \le 2$ (4). Din

(3) + (4) $\Rightarrow -5 \le 3a+2b \le 5 \Leftrightarrow \begin{cases} 3a+2b+5 \ge 0 \\ 3a+2b-5 \le 0 \end{cases} \Rightarrow \begin{cases} |3a+2b+5| = 3a+2b+5 \\ |3a+2b-5| = 5-3a-2b \end{cases}$ şi $N = 2a + 3b + 5 + 5 - 20 - 3b + 3a + 2b + 5 + 5 - 3a - 2b = 20$. **13.** a) $a^2+b^2 \ge 2ab \Leftrightarrow (a-b)^2 \ge 0$ (A); b) $a^2+b^2 \ge 2ab$, $b^2+c^2 \ge 2bc$, $c^2+a^2 \ge 2ac$ şi prin adunare obţinem inegalitatea cerută; c) $a+b \ge 2\sqrt{ab} \Leftrightarrow (a+b)^2 \ge 4ab \Leftrightarrow (a-b)^2 \ge 0$ (A); d) $\frac{a}{b}+\frac{b}{a} \ge 2 \Leftrightarrow a^2+b^2 \ge 2ab$. Egalitatea are loc pentru $a = b$;

e) $\frac{a+b}{c}+\frac{b+c}{a}+\frac{c+a}{b} \ge 6 \Leftrightarrow \frac{a}{c}+\frac{b}{c}+\frac{b}{a}+\frac{c}{a}+\frac{c}{b}+\frac{a}{b} \ge 6 \Leftrightarrow \left(\frac{a}{b}+\frac{b}{a}\right)+\left(\frac{b}{c}+\frac{c}{b}\right)+\left(\frac{c}{a}+\frac{a}{c}\right) \ge 6$,

din d) avem $\frac{a}{b}+\frac{b}{a} \ge 2$, $\frac{b}{c}+\frac{c}{b} \ge 2$, $\frac{c}{a}+\frac{a}{c} \ge 2$ şi prin adunarea membru cu membru a inegalităţilor se obţine concluzia; f) Presupunem că $\min(a, b) = a$ şi $\max(a, b) = b$, deci $a \le b$.

$a \le \frac{2ab}{a+b} \Leftrightarrow a^2+ab \le 2ab \Leftrightarrow a^2-ab \le 0 \Leftrightarrow a(a-b) \le 0$. Adevărat.

$\frac{2ab}{a+b} \le \sqrt{ab} \,\Big|: \sqrt{ab} \Leftrightarrow 2\sqrt{ab} \le a+b \Leftrightarrow 4ab \le (a+b)^2 \Leftrightarrow (a-b)^2 \ge 0$. Adevărat.

$\frac{a+b}{2} \le \sqrt{\frac{a^2+b^2}{2}} \Leftrightarrow \frac{(a+b)^2}{4} \le \frac{a^2+b^2}{2} \Leftrightarrow (a+b)^2 \le 2(a^2+b^2) \Leftrightarrow a^2+2ab+b^2 \le 2a^2+2b^2$ $\Leftrightarrow a^2-2ab+b^2 \ge 0 \Leftrightarrow (a-b)^2 \ge 0$. Adevărat.

$\sqrt{\frac{a^2+b^2}{2}} \le b \Leftrightarrow \frac{a^2+b^2}{2} \le b^2 \Leftrightarrow (a-b)(a+b) \le 0$. Egalitatea în inegalităţile mediilor are loc dacă şi numai dacă $a = b$; g) $x+\frac{1}{x} \ge 2 \Leftrightarrow \frac{x^2-2x+1}{x} \ge 0 \Leftrightarrow \frac{(x-1)^2}{x} \ge 0$. Adevărat.

14. a) Se aplică inegalităţile mediilor: $x+y \ge 2\sqrt{xy}$, $y+z \ge 2\sqrt{yz}$, $z+x \ge 2\sqrt{zx}$.
Prin înmulţirea membru cu membru a inegalităţilor, obţinem: $(x+y)(y+z)(z+x) \ge 8xyz$;

b) $x^2+\frac{y^2}{x^2} \ge 2\sqrt{x^2\cdot\frac{y^2}{x^2}} = 2y \Leftrightarrow x^2+\frac{y^2}{x^2} \ge 2y$; $y^2+\frac{z^2}{y^2} \ge 2\sqrt{y^2\cdot\frac{z^2}{y^2}} \Leftrightarrow y^2+\frac{z^2}{y^2} \ge 2z$;

$z^2+\frac{x^2}{z^2} \ge 2\sqrt{z^2\cdot\frac{x^2}{z^2}} \Leftrightarrow z^2+\frac{x^2}{z^2} \ge 2x$; Înmulţim membru cu membru inegalităţile şi obţinem: $\left(x^2+\frac{y^2}{x^2}\right)\left(y^2+\frac{z^2}{y^2}\right)\left(z^2+\frac{x^2}{z^2}\right) \ge 8xyz$; c) Rezultă din b). **15.** a) $(x+y+z)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) \ge 9 \Leftrightarrow$

$\Leftrightarrow \frac{x}{y}+\frac{y}{x}+\frac{y}{z}+\frac{z}{y}+\frac{x}{z}+\frac{z}{x} \geq 6$. b) Rezultă din a) pentru $x = y = z = 1$. **16.** a) este inegalitatea $m_g\ (a, b) \leq m_a(a, b)$. Egalitatea are loc pentru $a = b$; b) $\frac{\sqrt{2}}{3}=\frac{\sqrt{1\cdot 2}}{1+2}<\frac{1}{2}$; $\frac{\sqrt{6}}{5}=\frac{\sqrt{2\cdot 3}}{2+3}<\frac{1}{2}$; $\frac{\sqrt{12}}{7}=\frac{\sqrt{3\cdot 4}}{3+4}<\frac{1}{2}$; $\frac{\sqrt{20}}{9}=\frac{\sqrt{4\cdot 5}}{4+5}<\frac{1}{2}$; $\frac{\sqrt{30}}{11}=\frac{\sqrt{5\cdot 6}}{5+6}<\frac{1}{2}$; $\frac{\sqrt{42}}{13}=\frac{\sqrt{6\cdot 7}}{6+7}<\frac{1}{2}$ şi prin însumare rezultă inegalitatea cerută; c) $\frac{\sqrt{2}}{3}<\frac{1}{2}$, $\frac{\sqrt{6}}{5}<\frac{1}{2} \ldots \frac{\sqrt{2014\cdot 2015}}{4029}<\frac{1}{2}$ (conform inegalității dintre media geometrică şi media aritmetică). Prin adunarea acestor inegalități se obține inegalitatea cerută.

2.4. Inecuaţii de forma $ax + b > 0$ ($<, >, \leq, \geq$), $a, b \in \mathbb{R}$, $a \neq 0$ şi $x \in \mathbb{Z}$

1. a) da; b) nu; c) da; d) da; e) da; f) da. **2.** a) $S = \{0, 1, 2, 3\}$; b) $S = \{0, 1, 2, 3, 4\}$; c) $S = \varnothing$; d) $S = \{0, 1, 2, 3\}$; e) $S = \{0, 1, 2\}$; f) $S = \{0, 1, 2\}$. **3.** a) $A \cap S = \{-3; -2; -1; 0; 1; 2\}$; b) $A \cap S = \{1; 2; 3; 4; 5\}$; c) $A \cap S = \{-3; -2; -1\}$; d) $A \cap S = \{-3; -2; -1; 0; 1\}$; e) $A \cap S = \{0; 1; 2; 3; 4; 5\}$; f) $A \cap S = A$. **4.** a) $S = \{3, 4, 5, \ldots\}$; b) $S = \{\ldots, -1, 0, 1\}$; c) $S = \{3, 4, 5,\ldots\}$; d) $S = \mathbb{N}$; e) $S = \{\ldots, -10, -9, -8\}$; f) $S = \{7, 8, 9, \ldots\}$. **5.** a) $S = \{-2; -1; 0; 1, 2\}$; b) $S = \{0, 1, 2\}$; c) $S = \varnothing$; d) $S = \{2\}$; e) $S = \varnothing$; f) $S = \{-2, -1\}$. **6.** a) $x \geq -1$; b) $x \geq 2$; c) $\varnothing$; d) $x \leq 4$; e) $x \geq 4$; f) $x \leq 0$. **7.** a) $S = \{2, 3, 4, \ldots\}$; b) $S = \{2, 3, 4, \ldots\}$; c) $S = \mathbb{N}$; d) $x \leq 2$; e) $S = \{\ldots, -1; 0; 1\}$; f) $x \geq -2$. **8.** a) $x \in \{0; 1; 2; 3\}$; b) $x \in \{3\}$; c) $x \in \{-5; -4; -3; -2; -1; 0; 1; 2\}$; d) $x \in \{-5; -4; -3; -2; -1; 0\}$; e) $x \in \{-5; -4; -3; -2; -1\}$; f) $x \in \{1; 2; 3\}$. **9.** a) $x \leq 3$; b) $x < -5$; c) $x \geq 10$; d) $x \geq 14$; e) $x \leq -18$; f) $x < 5$. **10.** a) $x \geq 7$; b) $x \leq 12$; c) $x \leq 2$; d) $x \geq 3$; e) $x \leq 25$; f) $x \leq 2$. **11.** a) $x \in \{-3; -2; -1; 0; 1; 2\}$; b) $x \in \{-1; 0\}$; c) $x \in \{-3; -2; -1; 0; 1; 2; 4\}$; d) $x \in \{-2; -1; 0; 1; 2\}$; e) $x \in \{2; 3\}$; f) $x \in \{0; 1\}$. **12.** a) $x \geq 1$; b) $x \geq -1$; c) $x \leq -2$; d) condiție de existență: $x - 2 \neq 0 \Rightarrow x \neq 2$; inecuația devine: $\frac{\sqrt{3}-1-\sqrt{3}}{x-2}<0 \Leftrightarrow \frac{-1}{x-2}<0 \Rightarrow x-2>0 \Rightarrow x>2$; e) condiție de existență: $3x - 9 \neq 0 \Leftrightarrow x \neq 3$; inecuația devine $\frac{\sqrt{5}-2-\sqrt{5}-2}{3x-9} \leq 0 \Leftrightarrow \frac{-4}{3x-9} \leq 0 \Leftrightarrow 3x - 9 > 0 \Rightarrow x > 3$.

2.5. Probleme care se rezolvă cu ajutorul ecuaţiilor şi inecuaţiilor

1. 58, 42. **2.** 120, 80. **3.** 20000, 25000. **4.** 190, 230. **5.** 45, 25, 35. **6.** 4. **7.** 12 fete şi 18 băieți. **8.** 80, 48. **9.** 240, 180. **10.** 300, 250, 80. **11.** 200, 164, 276. **12.** 120, 160, 170, 190. **13.** 20, 7, 15. **14.** fiica = 6 ani, mama = 30 ani, tata = 36 ani. **15.** 1500 lei; **16.** 1000 lei; **17.** I: $\frac{1}{4}x + 10 \Rightarrow$ $\Rightarrow$ rest $\frac{3}{4}x-10$; II: $\frac{3x}{10}-4 \Rightarrow$ rest $=\frac{9x}{20}-6$; III: $\frac{3x}{20}+18 \Rightarrow$ rest $=\frac{3x}{10}-24 \Rightarrow \frac{3x}{10}-24=60 \Rightarrow$ $\Rightarrow \frac{3x}{10}=84 \Rightarrow 3x=840 \Rightarrow x=280$ km. **18.** 100; 50; 12; **19.** $\frac{3a}{2b}=\frac{4}{3} \Leftrightarrow 9a=8b \Leftrightarrow b=\frac{9a}{8}$;

$\frac{1}{2}\left(a+\frac{9a}{8}\right)+23=40 \Leftrightarrow \frac{1}{2}\cdot\frac{17a}{8}=17 \Leftrightarrow a=16,\ b=18$. **20.** 24 lei, 20 lei.

2.6. Probleme pentru olimpiade şi concursuri

1. Avem $\frac{x+1}{x}=\frac{y+3}{y}=\frac{z+6}{z} \Leftrightarrow \frac{1}{x}=\frac{3}{y}=\frac{6}{z}=18:3=6$. Atunci $x=\frac{1}{6},\ y=\frac{1}{2},\ z=1$ şi $x+y-$ $-2z=\frac{1}{6}+\frac{1}{2}-2=-\frac{4}{3}$. **2.** Avem $100 \le \overline{abc}=(x-2)(x-1)\le 999$, deci $12 \le x \le 33$. Suma este $\frac{1}{10\cdot 11}+\frac{1}{11\cdot 12}+...+\frac{1}{32\cdot 33}=\frac{1}{10}-\frac{1}{11}+\frac{1}{11}-\frac{1}{2}+...+\frac{1}{32}-\frac{1}{33}=\frac{23}{330}$. **3.** Din $\frac{2x}{3x+1}=\frac{4y}{6y+2}$ rezultă $x = 2y$. Din $\frac{2x}{3x+1}=\frac{x+z}{z+7}$ rezultă $\frac{2x}{3x+1-2x}=\frac{x+z}{z+7-x-z}=\frac{x+z}{7-x}=\frac{2x+2z}{14-2x}=$ $=\frac{2x+2z-2x}{14-2x-x-1}=\frac{2z}{13-3x}$, de unde $z=\frac{13x-3x^2}{x+1}=-3x+10-\frac{10}{x+1}\in\mathbb{Z} \Rightarrow x \in \{\pm 10;\ \pm 5;\ \pm 1\}$ etc. **4.** Pentru $c = 2015$ avem $a + b = 0$ şi deci avem soluția (0, 0, 2015). Pentru $c = 2014$ avem $a + b = 1$ şi deci avem soluțiile (0, 1, 2014), (1, 0, 2014). În total avem $1 + 2 + 3 +\ldots+ 2016 =$ $= 1008 \cdot 2017$ soluții. **5.** Fie $\frac{x}{z+1}=\frac{y}{z+4}=\frac{z}{3}=\frac{y-x}{3}=a\in\mathbb{Q}$. Avem $z = 3a$, $y = 3a^2 + 4a$, $x =$ $= 3a^2 + a$, $y = x + z$. a) Evident că dacă 2 din numerele x, y, z sunt întregi, cum $y = x + z$, atunci şi al treilea număr este întreg; b) Dacă $y=\frac{x+z}{2}$ avem $2y = y$ şi $y = 0$ (fals). Dacă $x + y = 2z$ avem soluția (2, 4, 2). Dacă $z + y = 2x$ avem $y = 3x$, $x = 2z$ etc. **6.** $S_n=\left(\sqrt{2}-\sqrt{1}\right)+\left(\sqrt{3}-\sqrt{2}\right)+$ $+ ... + \left(\sqrt{n}-\sqrt{n-1}\right)=\sqrt{n}-1\in\mathbb{N}^* \Leftrightarrow n=\left(m+1\right)^2, m\in\mathbb{N}^*$. **7.** $\sum_{k=m}^{n}\frac{\sqrt{k+1}-\sqrt{k}}{\sqrt{k(k+1)}}=\sum_{k=m}^{n}\left(\frac{1}{\sqrt{k}}-\frac{1}{\sqrt{k+1}}\right)=$ $=\frac{1}{\sqrt{m}}-\frac{1}{\sqrt{n+1}}=\frac{7}{30}=\frac{1}{3}-\frac{1}{10}=\frac{1}{\sqrt{9}}-\frac{1}{\sqrt{100}} \Rightarrow m = 9;\ n = 99 \Rightarrow B=\left\{\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n^2+n}}\middle| n=\overline{9;99}\right\}$.
8. $3=\left||2-x|+|y-3x|\right|=|x-2|+|y-3x|$. Avem cazurile: a) $x = 2$, $|y - 6| = 3 \Rightarrow (x, y) \in \{(2, 9);$ $(2, 3)\}$; b) $|x-2|=1,\ |y-3x|=2\Rightarrow(x,y)\in\{(1,5);(1,1);(3,11);(3,7)\}$; c) $|x-2|=2$, $|y-3x|=$ $= 1\Rightarrow(x,y)\in\{(0,1);(0,-1);(4,11);(4,13)\}$; d) $|x-2|=3$, $|y-3x|=0\Rightarrow(x,y)\in\{(-1,-3);(5,15)\}$.
9. $(x^2-3)(y^2-3)=22 \Rightarrow$ avem cazurile: a) $x^2 - 3 = 22$, $y^2 - 3 = 1 \Rightarrow (x, y) \in\{(5, 2); (5, -2);$ $(-5, -2); (-5, 2)\}$; b) $x^2 - 3 = 1$, $y^2 - 3 = 22 \Rightarrow (x, y) \in \{(2, 5); (-2, 5); (-2, -5); (2, -5)\}$; c) $x^2 - 3 = -1 \Rightarrow x \notin \mathbb{Z}$; d) $x^2 - 3 = -22 \Rightarrow x \notin \mathbb{Z}$. **10.** $1+m\sqrt{2}=2a+na\sqrt{2}$, $a, m, n \in \mathbb{Q} \Rightarrow$ $\Rightarrow 2a = 1$, $m = na \Rightarrow n = 2\cdot m$. Din $n^2 + m^2 = 5m^2 \le 45 \Rightarrow |m| \le 3 \Rightarrow \text{card}C = 7$; **11.** Notăm $x - 2014 = a$. Avem $\left(\frac{a}{2013}+1\right)+\left(\frac{a}{2010}+1\right)+\left(\frac{a}{2007}+1\right)=\left(\frac{a}{7}+1\right)+\left(\frac{a}{10}+1\right)+\left(\frac{a}{13}+1\right)$, de unde rezultă $a = 0$ şi deci $x = 2014$. **12.** Pentru $m = 2p$, $n =2p + 1$, $p \in \mathbb{N}$ avem $a=\sqrt{3^{2p}\cdot 16}=$ $=4\cdot 3^p$. **13.** a) 143; b) $\overline{abcd}\in\{1694;\ 9614\}$. **14.** a) (1, 6), (6, 1); b) Observăm că $x, y, z \in\mathbb{N}^*$ –

$-\{1\}$. Presupunem $2 \le x \le y \le z$ şi deci $\frac{1}{x} \ge \frac{1}{y} \ge \frac{1}{z} \Rightarrow \frac{3}{x} \ge \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 \Rightarrow x \le 3$. Dacă $x = 2$ rezultă $yz = 2y + 2z \Leftrightarrow (y-2)(z-2) = 4 \Rightarrow$ soluţiile (2, 4, 4), (2, 3, 6), (2, 6, 3). Dacă $x = 3$ rezultă $(2y-3)(2z-3) = 9$ şi avem soluţiile (3, 6, 2), (3, 2, 6), (3, 3, 3) etc.; c) (2, 3, 6) şi permutările; d) Presupunem $0 \le x \le y \le z$. Avem soluţiile (0, 0, 2), (0, 1, 1), (1, 1, 1) şi permutările acestora. Presupunem $x \ge 2$. Avem $xyz + 2 - x - y - z = x(yz-1) + 2 - y - z \ge 2(yz - 1) + 2 - y - z = 2yz - y - z = y(2z-1) - z \ge 2(2z-1) - z = 3z - 2 > 0$ şi nu mai avem soluţii. e) Deoarece $2 \mid x^2 + x - 32$ rezultă $2 \mid y$. Cum $x^2 + x = x(x+1) \le 32$, rezultă $x \in \{0, 1, 2, 3, 4, 5\}$. Obţinem soluţiile (1, 10), (4, 4); f) Cum $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} \ge 2$ şi $\frac{1}{x} + \frac{1}{y} \le 2$, obţinem $x = y = 1$; g) $(x-1)^2 + (y-2)^2 + z = 2 \Rightarrow z \in \{0, 1, 2\} \Rightarrow$ soluţiile (1, 2, 2), (2, 2, 1), (0, 2, 1), (1, 1, 1), (1, 3, 1), (2, 3, 0), (2, 1, 0), (0, 3, 0), (0, 1, 0); h) $n(n+1) = 2\sqrt{\overline{abc}} \in [20; 62] \Rightarrow (n, \overline{abc}) \in \{(4,100), (5,225), (6,441)\}$; **15.** a) $(2x-1)(y-2) = 0 \Rightarrow x \in \mathbb{Z}, y = 2$; b) $xy(x-y)(x+y) = 2(x-y)$; Dacă $x \ne y$ avem $xy(x+y) = 2$ şi atunci avem soluţiile (1, 1), (1, –2), (–2, 1), (–1, –1); c) $x^2y^2 - (x-3)^2 = 0 \Leftrightarrow (xy - x + 3)(xy + x - 3) = 0$. Mulţimea de soluţii este (1, –2), (–1, 4), (3, 0), (–3, 2), (1, 2), (–1, –4); d) Avem $(x^2y^2 + 1) + (x^2 + y^2) \ge 2|xy| + 2|xy| = 4|xy| \ge 4xy$. Avem „=” pentru $x = y \in \{-1, 1\}$; e) Orice pătrat perfect este de forma $\mathcal{M}_4$ sau $\mathcal{M}_4 + 1$. Deci $-x^2 + y^2$ este de forma $\mathcal{M}_4$, $\mathcal{M}_4 + 1$ sau $\mathcal{M}_4 + 3$. Cum $2014 = \mathcal{M}_4 + 2$, ecuaţia nu are soluţie. **16.** a) $(x^2 - 5x + 4)(x^2 - 5x + 6) = 24 \Leftrightarrow (x^2 - 5x)^2 + 10(x^2 - 5x) = 0 \Leftrightarrow (x^2 - 5x)(x^2 - 5x + 10) = 0 \Leftrightarrow x \in \{0; 5\}$; b) $(x^2 - 9)(x^2 - 1) = 105 \Leftrightarrow x^4 - 10x^2 = 96 \Leftrightarrow (x^2 - 5)^2 = 121 \Leftrightarrow (x^2 - 16) \cdot (x^2 + 6) = 0 \Leftrightarrow x \in \{\pm 4\}$. **17.** Avem $12 = (x + y) + \left(\frac{x\sqrt{x}}{\sqrt{y}} + \frac{y\sqrt{y}}{\sqrt{x}}\right) \ge 2\sqrt{xy} + 2\sqrt{\frac{x\sqrt{x} \cdot y\sqrt{y}}{\sqrt{x} \cdot \sqrt{y}}} = 4\sqrt{xy} \le 3 \Rightarrow xy \le 9$. Avem $\max(xy) = 9$ pentru $x = y = 3$. **18.** a) $\sqrt{x-y} > \sqrt{x} - \sqrt{y} \Leftrightarrow x - y > x + y - 2\sqrt{xy} \Leftrightarrow \sqrt{x} > \sqrt{y}$ (adunarea); b) $S = \sum_{k=1}^{60} \sqrt{\frac{(2k+1)(2k-1)}{(2k-1)(2k+1)}} = \sum_{k=1}^{60} \sqrt{\frac{1}{2k-1} - \frac{1}{2k+1}} > \sum_{k=1}^{60} \left(\frac{1}{2k-1} - \frac{1}{2k+1}\right) = \frac{12}{121}$. **19.** Pentru $n \in \{1, 2, 3\}$ avem $n^n = n^4 + 3n^2 - 3n^3$. Pentru $n \ge 4$ avem $n^4 + 3n^2 - 3n^3 = n^4 - 3n^2(n-1) < n^4 \le n$. **20.** Avem $S = \frac{1}{a + \frac{4}{a}} + \frac{1}{b + \frac{4}{b}} + \frac{1}{c + \frac{4}{c}} = \frac{a}{a^2 + 4} + \frac{b}{b^2 + 4} + \frac{c}{c^2 + 4}$. Cum $\frac{a}{a^2 + 4} \le \frac{1}{4}$ şi analoagele avem $S \le \frac{3}{4}$. Avem „=” dacă şi numai dacă $a = b = c = 2$ (fals).

TESTE DE EVALUARE

Testul 1

I. 1. $\frac{16}{3}$. **2.** $S = \{-3\sqrt{3}, 3\sqrt{3}\}$. **3.** –2. **4.** $S = \{-2; -1\}$. **5.** $S = \{0, 1, 2\}$. **6.** –4.

II. 1. a) $10x + 10x - 35 - 6x - 2 = 50 - 5x - 30 \Leftrightarrow 19x = 57 \Leftrightarrow x = 3$; b) $2x\left(2+\sqrt{3}\right) - x\left(\sqrt{3}-1\right) -$

$-2x = 8 + 2\left(\sqrt{3}-1\right) \Leftrightarrow x\left(4+2\sqrt{3}-\sqrt{3}+1-2\right) = 6+2\sqrt{3} \Leftrightarrow x\left(3+\sqrt{3}\right) = 2\left(3+\sqrt{3}\right) \Leftrightarrow x = 2$.

2. $190 - 60x - 14x + 14 \geq 35x - 98x + 168 \Leftrightarrow -11x \geq -36 \Leftrightarrow x \leq \dfrac{36}{11} \Leftrightarrow x \in \{...,1, 2, 3\}$. **3.** x = = numărul răspunsurilor greşite. $(20 - x) \cdot 3 - 2x = 35 \Leftrightarrow 60 - 5x = 35 \Leftrightarrow 25 = 5x \Leftrightarrow x = 5$.

Testul 2

I. 1. 4; **2.** $S = \{0, 1, 2, 3\}$. **3.** –1. **4.** –4, –3, –2, –1. **5.** $\sqrt{2}-1$. **6.** 6.

II. 1. $3x + 18 - 4x^2 - 4x - 1 - 13 < x^2 - 4x + 4 - 10x - 5x^2 \Leftrightarrow x < 0 \Leftrightarrow x \in \{\ldots, -3, -2, -1\}$.
2. a) $3x + 4 - 2x + 2 = 4x - 12 + 18 - 3x \Leftrightarrow 0 \cdot x = 0 \Rightarrow S = \mathbb{R}$; b) $mx - 2mx - x = 1 - 2 - m \Leftrightarrow$

$$\Leftrightarrow \left.\begin{array}{l} x(-m-1) = -m-1 \\ S = \mathbb{R} \end{array}\right\} \Rightarrow m = -1 \text{. } \mathbf{3.}\ a \cdot \frac{2}{3} = b \cdot \frac{2}{5} \Leftrightarrow \frac{a}{3} = \frac{b}{5} = k = \begin{cases} a = 3k \\ b = 5k \end{cases}$$

$$\left.\begin{array}{l} \left.\begin{array}{l} a \cdot b > 45 \Leftrightarrow 15k^2 > 45 \Leftrightarrow k^2 > 3 \\ k \in \mathbb{N} \end{array}\right\} k \geq 2 \\ a + b < 20 \Leftrightarrow 8k < 20 \Leftrightarrow k < \dfrac{5}{2} \end{array}\right\} k = 2 \Rightarrow \begin{cases} a = 6 \\ b = 10 \end{cases}$$

Testul 3

I. 1. $-\dfrac{2}{3}$. **2.** 2. **3.** $p = \dfrac{4}{7}$; **4.** 1,85. **5.** $\{0; 1\}$. **6.** 4.

II. 1. a) $3 - 6x + 3x^2 - 4x^2 + 16x - 16 + 5x^2 - 30x + 45 = 4x^2 - 12x + 9 + 4 - x \Leftrightarrow -7x = -19 \Leftrightarrow$ $\Leftrightarrow x = \dfrac{19}{7}$; b) $(2x + 3)(x + 4) + (x + 2)(x - 4) = 2(x^2 - 16) + x(x - 3) \Leftrightarrow 9x + 4 = -3x - 32 \Leftrightarrow x =$ $= -3$. **2.** $12 - 15\sqrt{2}x + 8\sqrt{2}x + 7\sqrt{2} < 12x - 8\sqrt{2} + 2\sqrt{2} - 12 - 3\sqrt{2} \Leftrightarrow -9\sqrt{2}x - 12x < -15\sqrt{2} -$ $-24 - 3\sqrt{2} \Leftrightarrow -3x\left(3\sqrt{2}+4\right) < -6\left(3\sqrt{2}+4\right) \Leftrightarrow x > 2 \Rightarrow x \in \{3, 4, 5,\ldots\}$. **3.** x – prețul inițial;
$\dfrac{4x}{5} + \dfrac{30}{100} \cdot \dfrac{4x}{5} = 780 \Leftrightarrow \dfrac{26x}{25} = 780 \Leftrightarrow x = \dfrac{780 \cdot 25}{26} \Leftrightarrow x = 30 \cdot 25 \Leftrightarrow x = 750$ lei.

CAPITOLUL 3
Elemente de organizare a datelor

3.3. Probabilitatea realizării unor evenimente

1. $\dfrac{3}{6} = \dfrac{1}{2}$. **2.** a) $\dfrac{2}{5}$; b) $\dfrac{3}{5}$; c) 1; d) 0. **3.** a) $\dfrac{4}{5}$; b) $\dfrac{5}{9}$. **4.** a) $\dfrac{99}{100}$; b) $\dfrac{1}{100}$. **5.** a) $\dfrac{1}{10}$; b) $\dfrac{1}{25}$; c) $\dfrac{3}{25}$; d) $\dfrac{9}{100}$. **6.** $\dfrac{5}{18}$. **7.** a) $\dfrac{2}{5}$; b) $\dfrac{3}{5}$; c) $\dfrac{13}{20}$. **8.** $\dfrac{7}{36}$. **9.** $\dfrac{1}{3}$. **10.** $\dfrac{27}{100}$. **11.** $\dfrac{1}{6}$. **12.** sunt 33 de numere divizibile cu 3, 25 de numere divizibile cu 4 şi 8 numere divizibile cu 12, deci sunt în total 33 + 25 – 8 = 50 numere divizibile cu 3 sau 4 deci $p = \dfrac{50}{100} = \dfrac{1}{2}$. **13.** sunt 51 de numere divizibile cu 2; 34 numere divizibile cu 3 şi 17 numere divizibile cu 6, deci în total sunt

$51 + 34 - 17 = 68$ numere divizibile cu 2 sau 3, deci $P = \frac{68}{101}$. **14.** $P = \frac{2012}{2016} = \frac{1006}{1008} = \frac{503}{504}$. **15.** sunt 9 numere cu ambele cifre egale deci $90 - 9 = 81$ numere au cifrele distincte de unde rezultă $p = \frac{81}{90} = \frac{9}{10}$. **16.** numere de patru cifre sunt de forma $\overline{abcd}$; pentru că produsul cifrelor să fie impar este necesar ca toate cifrele numărului să fie impare deci $a, b, c, d \in \{1, 3, 5, 7, 9\}$ deci 5^4 cazuri favorabile din 9000 de cazuri posibile deci $p = \frac{5^4}{9000}$. **17.** $a + r + v = 45$; $\frac{a}{45} = \frac{1}{3} \Rightarrow a = 15$; $\frac{r}{45} = \frac{4}{15} \Rightarrow r = 12$; $\frac{v}{45} = \frac{2}{5} \Rightarrow v = 18$. **18.** numărul 51 este de forma $4k + 3$ şi nu există pătrate perfecte de această formă, deci $p = 0$. **19.** dacă a = este cifră atunci $u(a^3) = u(a) \Leftrightarrow a \in \{0, 1, 4, 5, 6, 9\}$, deci sunt $9 \cdot 6 = 54$ cazuri favorabile de unde $p = \frac{3}{5}$. **20.** m şi n trebuie să fie divizibile cu 6, deci $m, n \in \{6; 12; 18; 24; 30; 36\}$ deci numărul cazurilor favorabile este $6 \cdot 6 = 36$ în timp ce numărul cazurilor posibile este 40^2, deci $p = \frac{6^2}{40^2} = \left(\frac{3}{20}\right)^2 = \frac{9}{400}$. **21.** Numerele de 5 cifre sunt de forma $\overline{abcde}$ cu a, b, c, d, e, cifre, unde $a \in \{1, 2, 3, 4, 6, 7, 8, 9\}$, $b, c, d, e \in \{0; 1; 2; 3; 4; 6; 7; 8; 9\}$ deci $8 \cdot 9^4$ numere de 5 cifre nu conțin cifra 5, deci $p = \frac{8 \cdot 9^4 + 1}{10^6}$. **22.** 7^n este de forma $4k + 1$ pentru n par şi respectiv de forma $4k + 3$ pentru n impar, iar $u(7^{4k+1}) = 7$; $u(7^{4k+3}) = 3$, de unde obținem $u(7^{4k+1} + 2) = 9$ şi respectiv $u(7^{4k+3}) = 5$, deci $p = \frac{500}{1000} = \frac{1}{2}$; **23.** $a + b + c = 13$ de unde se obțin 21 de cazuri favorabile în timp ce numărul cazurilor posibile este 216, deci $p = \frac{21}{216} = \frac{7}{72}$. **24.** a) negre + albastre = 30; albe + + albastre = 42; albe + negre = 54 deci în urmă sunt 63 de bile: 33 albe; 21 negre şi 9 albastre; b) $p_{\text{alb}} = \frac{11}{21}$; $p_{\text{negru}} = \frac{1}{3}$; $p_{\text{albastru}} = \frac{1}{7}$; c) $p = \frac{9}{63} \cdot \frac{21}{62} = \frac{1}{7} \cdot \frac{21}{62} = \frac{3}{62}$.

GEOMETRIE

CAPITOLUL 4
Relaţii metrice în triunghiul dreptunghic

4.2. Teorema înălţimii

1. $AD^2 = BD \cdot DC \Leftrightarrow 81 = 3 \cdot DC \Leftrightarrow DC = 27$ cm şi $BC = BD + DC \Leftrightarrow BC = 3 + 27 \Rightarrow BC = 30$ cm. **2.** $DC = BC - BD \Leftrightarrow DC = 8$ cm; $AD^2 = BD \cdot DC \Leftrightarrow AD^2 = 2 \cdot 8 \Leftrightarrow AD = 4$ cm. **3.** $AD^2 = BD \cdot DC \Leftrightarrow 144 = BD \cdot 8 \Leftrightarrow BD = 18$ cm şi $BC = 26$ cm. **4.** $AD^2 = \frac{25}{4} \cdot \frac{49}{9} \Rightarrow AD = \frac{5}{2} \cdot \frac{7}{3} \Rightarrow AD = \frac{35}{6}$ cm $\Rightarrow AD = 5{,}8(3)$ cm. **5.** $CD = 3BD \Rightarrow BC = 4BD$, 15 cm $= 4BD \Rightarrow BD = \frac{15}{4}$ cm $\Rightarrow CD = \frac{45}{4}$ cm, $AD = \sqrt{CD \cdot BD} = \sqrt{\frac{45}{4} \cdot \frac{15}{4}} = \frac{15\sqrt{3}}{4}$ cm. **6.** $\frac{CD}{1{,}5} = \frac{BD}{0{,}(6)} = k \Rightarrow CD = \frac{3}{2}k$, $BD = \frac{2}{3}k$; $AD^2 = CD \cdot BD \Rightarrow k = 12 \Rightarrow CD = 18$ dm şi $BD = 8$ dm $\Rightarrow BC = 26$ dm. **7.** $3BD + 3CD = 60$ dm; $4BD - 3CD = 3$ dm, deci $7BD = 63$ dm $\Rightarrow BD = 9$ dm, $CD = 11$ dm, $AD = \sqrt{BD \cdot DC} \Rightarrow AD = 3\sqrt{11}$ dm. **8.** Fie $CE \perp AB$, $E \in AB$. $AE = DC = 36$ cm $\Rightarrow EB = 16$ cm. $CE^2 = AE \cdot EB \Rightarrow CE = \sqrt{36 \cdot 16} = 24$ cm $\Rightarrow AD = 24$ cm, $\mathscr{A}_{ABCD} = 1056$ cm^2. **9.** Fie $F = pr_{BC}O$. $[OF]$ linie mijlocie în $\Delta BED \Rightarrow OF = 6$ cm; $pr_{BC}[BO] = [BF]$. În ΔBOC, $OF^2 = BF \cdot FC \Rightarrow 36 = 9 \cdot FC \Rightarrow FC = 4$ cm $\Rightarrow BC = 13$ cm. **10.** $AB + CD = 96$ cm şi $AB - CD = 24$ cm $\Rightarrow AB = 60$ cm şi $CD = 36$ cm. Fie $DE \perp AB$, $E \in AB$. Avem $AE = 12$ cm şi $EB = 48$ cm. $DE^2 = AE \cdot EB$ deci $DE = \sqrt{12 \cdot 48} = 24$ cm. $\mathscr{A}_{ABCD} = \frac{(60+36) \cdot 24}{2} = 1152$ cm^2. **11.** $351 = \frac{BC \cdot 18}{2} \Rightarrow BC = 39$ cm. Fie $AD \perp BC$, $D \in BC$. $\frac{BD}{9} = \frac{DC}{4} = k \Rightarrow BD = 9k$, $DC = 4k$, $13k = 39 \Rightarrow k = 3 \Rightarrow BD = 27$ cm, $DC = 12$ cm, $BD \cdot DC = AD^2 \Leftrightarrow 27 \cdot 12 = 18^2 \Leftrightarrow 324 = 324 \Rightarrow m(\sphericalangle BAC) = 90°$. **12.** $\frac{BE}{DE} = \frac{9}{16} \Leftrightarrow \frac{BE}{9} = \frac{DE}{16} = k \Rightarrow BE = 9k$, $DE = 16k$. $CE^2 = BE \cdot DE \Leftrightarrow 48^2 = 9 \cdot 16k^2 \Rightarrow k = 4 \Rightarrow BE = 36$ cm, $DE = 64$ cm $\Rightarrow BD = 100$ cm. $\mathscr{A}_{ABCD} = 2\mathscr{A}_{\Delta BCD} = BD \cdot CE = 4800$ cm^2. **13.** $AC = 2DC \Rightarrow m(\sphericalangle CAD) = m(\sphericalangle ABC) = 30°$ (1); $m(\sphericalangle CAB) = 60° \Rightarrow m(\sphericalangle ACB) = 90°$ (2). Din (1), (2) $\Rightarrow AB = 32$ cm. Fie $CE \perp AB$, $E \in AB$; $CE^2 = AB \cdot EB \Rightarrow CE = \sqrt{8 \cdot 24} \Rightarrow CE = 8\sqrt{3}$ cm; $\mathscr{A}_{ABCD} = \frac{(32+8) \cdot 8\sqrt{3}}{2} = 160\sqrt{3}$ cm^2. **14.** $\frac{AD \cdot BD}{2} \cdot \frac{DC \cdot AD}{2} = 64 \Rightarrow AD^4 = 4^4 \Rightarrow AD = 4$ cm. $\mathscr{A}_{ABC} = \frac{BC \cdot AD}{2} \Rightarrow \mathscr{A}_{ABC} = 24$ cm^2. **15.** $DE = \frac{AD}{2} = \frac{BD}{4} \Rightarrow BE = \frac{3BD}{4}$; $AE = \frac{AB \cdot AD}{BD} \Leftrightarrow AE = \frac{32\sqrt{3}}{BD}$. Conform teoremei înălțimii avem $AE^2 = BE \cdot DE$; $AE^2 = \frac{3BD^2}{16} \Leftrightarrow$

$\Leftrightarrow \left(\dfrac{32\sqrt{3}}{BD}\right)^2 = \dfrac{3BD^2}{16} \Leftrightarrow 3BD^4 = 2^4 \cdot 2^{10} \cdot 3 \Leftrightarrow BD^4 = 2^{14} \Leftrightarrow BD^2 = 2^7 \Leftrightarrow BD = 8\sqrt{2} \Leftrightarrow DE =$ $= \dfrac{8\sqrt{2}}{4} = 2\sqrt{2}$.

4.3. Teorema catetei

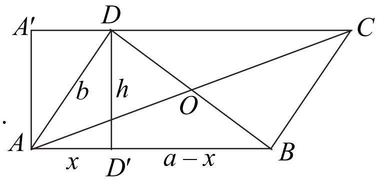
1. $DC = 9$ cm; $AB^2 = BD \cdot BC \Rightarrow AB = 20$ cm; $AC^2 = CD \cdot BC \Rightarrow AC = 15$ cm; $AD^2 = BD \cdot DC \Rightarrow$ $\Rightarrow AD = 12$ cm. **2.** $BD = \dfrac{AB^2}{BC} \Rightarrow BD = 3\sqrt{3}$ cm. $DC = 9\sqrt{3}$ cm; $AC^2 = CD \cdot CB \Rightarrow AC = 18$ cm; $AD^2 = BD \cdot DC \Rightarrow AD = 9$ cm. **3.** $BC = BD + DC \Rightarrow BC = 64$ cm; $AD = \sqrt{BD \cdot DC} \Rightarrow AD =$ $= 16\sqrt{3}$ cm; $\mathscr{A}_{\Delta ABC} = \dfrac{BC \cdot AD}{2} \Rightarrow \mathscr{A}_{\Delta ABC} = \dfrac{64 \cdot 16\sqrt{3}}{2} = 512\sqrt{3}$ cm^2; $AB = \sqrt{BD \cdot DC} \Rightarrow AB =$ $= 32$ cm; $AC = \sqrt{48 \cdot 64} = 32\sqrt{3}$ cm; $\mathscr{P}_{ABC} = 32\left(3 + \sqrt{3}\right)$ cm. **4.** $ND = 24$ cm; $MN = \dfrac{DN^2}{NE} \Rightarrow MN =$ $= 36$ cm; $\mathscr{P}_{MNP} = MN + NP + MP = 120$ cm. $ME = 20$ cm; $DE = \sqrt{ME \cdot EN} \Rightarrow DE = 8\sqrt{5}$ cm. $\mathscr{A}_{\Delta MDN} = \dfrac{MN \cdot DE}{2} \Rightarrow \mathscr{A}_{\Delta MDN} = \dfrac{36 \cdot 8\sqrt{5}}{2} = 144\sqrt{5}$ cm$^2 \Rightarrow \mathscr{A}_{\Delta MNP} = 288\sqrt{5}$ cm^2. **5.** $AD^2 = AE \cdot$ $\cdot AB \Rightarrow AB = 80$ cm; $\mathscr{P}_{ABCD} = 2(AB + AD) = 2(80 + 40) = 240$ cm. $BE = 60$ cm; $DE^2 = AE \cdot EB \Rightarrow$ $\Rightarrow DE = \sqrt{20 \cdot 60} = 20\sqrt{3}$ cm; $\mathscr{A}_{ABCD} = AB \cdot DE \Rightarrow \mathscr{A}_{ABCD} = 80 \cdot 20\sqrt{3} = 1600\sqrt{3}$ cm^2. **6.** $AE^2 =$ $= DE \cdot DB \Rightarrow AD = 2\sqrt{13}$ cm; $AB^2 = BE \cdot BD \Rightarrow AB = 3\sqrt{13}$ cm; $\mathscr{P}_{ABCD} = 10\sqrt{13}$ cm. **7.** Fie $CE \perp AD$, $E \in AD$; $CD^2 = AD \cdot DE \Rightarrow DE = 18$; $AE = 14$; $AC = 8\sqrt{7}$; $\mathscr{P} = 70 + 6\sqrt{7}$. **8.** $BD =$ $= 28$ cm; $CD = 12$ cm; $AB = 4\sqrt{70}$ cm; $AC = 4\sqrt{30}$ cm; $AD = 4\sqrt{21}$ cm. **9.** Fie $OM \perp AD$; $DM = \dfrac{9}{5}$ cm $\Rightarrow AM = \dfrac{16}{5}$ cm $\Rightarrow AO = 4$ cm $\Rightarrow AC = 8$ cm $\Rightarrow \mathscr{A}_{ABCD} = 24$ cm^2. **10.** $\mathscr{P}_{ABCD} =$ $= 4\left(5\sqrt{2} + 2 + \sqrt{6}\right)$ cm; $\mathscr{A}_{ABCD} = 80\sqrt{2}$ cm^2.

4.4. Teorema lui Pitagora

1. $BC = 10$ cm, $b = 4{,}8$ cm; $pr_{BC}AB = 3{,}6$ cm, $pr_{BC}AC = 6{,}4$ cm. **2.** 6 cm. **3.** 15 cm. **4.** 25 cm. **5.** 12 cm. **6.** $4\sqrt{3}$ cm. **7.** 72 cm. **8.** $10\sqrt{2}$ cm. **9.** 80 cm. **10.** a) da; b) da; c) da; d) da; e) da; f) nu. **11.** $10\sqrt{3}$ cm şi 10 cm. **12.** $24\left(\sqrt{3} + 1\right)$ cm. **13.** a) $AD = 20$ cm; b) $BE = 24$ cm; $CE = 18$ cm. **14.** a) $AC = 15$ cm; b) 7,2 cm. **15.** a) $AB = 6\sqrt{5}$ cm; $AC = 12\sqrt{5}$ cm; $BC = 30$ cm; b) $AD = 12$ cm. **16.** a) $AB = 12$ cm; $AC = 16$ cm; $BC = 20$ cm; b) $AD = 9{,}6$ cm. **17.** $AB = 4\sqrt{13}$ cm; $AC = 6\sqrt{13}$ cm; $BC = 26$ cm. **18.** a) $AC = 12\sqrt{2}$ cm; b) $MNPQ$ este pătrat, $\mathscr{P}_{MNPQ} = 16\sqrt{5}$ cm. **19.** a) $AB = 18$ cm; b) $6\sqrt{10}$ cm. **20.** $b = 12$ cm, $d = 3\sqrt{41}$ cm. **21.** a) $\mathscr{P}_{ABCD} = 120$ cm; $AC =$ $= 24\sqrt{3}$ cm; b) se aplică reciproca teoremei lui Pitagora; c) $576\sqrt{3}$ cm^2. **22.** $\mathscr{P}_{ABCD} =$ $= 6\left(7 + \sqrt{3}\right)$ cm; $AC = 6\sqrt{7}$ cm; $BD = 12\sqrt{3}$ cm. **23.** $\ell = 13$ cm; $h = \dfrac{120}{13}$ cm. **24.** $AB = 24$ cm;

$AC = 24$ cm. **25.** a) $\mathscr{A} = 900$ cm^2; $\mathscr{P}_{ABCD} = 12\left(\sqrt{26}+5\right)$ cm; b) $d = 30\sqrt{2}$ cm; c) $\mathscr{A} = 1620$ cm^2. **26.** a) 276 cm^2; b) $AC = 3\sqrt{65}$ cm, d(B, AC) $= \dfrac{24\sqrt{65}}{13}$ cm. **27.** a) $\mathscr{P}_{ABCD} = 90$ cm; $AC = 18\sqrt{3}$ cm; b) $\mathscr{A}_{ABCD} = 243\sqrt{3}$ cm^2; c) d(O, AD) $= 6\sqrt{3}$ cm; d(O, BC) $= 3\sqrt{3}$ cm. **28.** a) $AB = 30$ cm; b) d(D, AB) = 14,4 cm; c) $AC = 6\sqrt{73}$ cm. **29.** a) $\mathscr{P}_{ABCD} = 2\left(19+\sqrt{61}\right)$ cm; b) $AC = 4\sqrt{13}$ cm; $BD = 6\sqrt{13}$ cm; c) se exprimă aria triunghiului ΔABC în două moduri: d(A, BC) $= \dfrac{108\sqrt{61}}{61}$ cm. **30.** a) $BD = 12$ cm; b) $AB = 6\sqrt{3}$ cm; $AD = 6$ cm; m($\sphericalangle AC, BD$) = 60°, $\mathscr{P}_{ABCD} = 12\left(\sqrt{3}+1\right)$ cm; $\mathscr{A}_{ABCD} = 36\sqrt{3}$ cm^2; c) $CM = 3\sqrt{7}$ cm. **31.** Fie $AD \perp BC$. Notăm $BD = x \Rightarrow CD = 14 - x$. Aplicăm teorema lui Pitagora în triunghiurile dreptunghice ΔADB, respectiv ΔADC şi obţinem $AD^2 = 169 - x^2$ şi respectiv $AD^2 = 15^2 - (14 - x) \Rightarrow 169 - x^2 = 225 - 196 + 28x - x^2 \Rightarrow x = 5 \Rightarrow AD = 12$ cm. **32.** Da, m($\sphericalangle BAC$) = 90°. **33.** a) $\mathscr{P}_{ABCD} = \left(7\sqrt{7}+5\sqrt{3}+10\right)$ cm; $\mathscr{A}_{ABCD} = \left(42+5\sqrt{3}\right)$ cm^2; b) trapez, deoarece $AM \perp BD$, $CN \perp BD \Rightarrow AM \parallel CN$, $AM = \dfrac{12\sqrt{7}}{5}$ cm; $CN = \dfrac{10\sqrt{21}}{7}$ cm, $AM \neq CN \Rightarrow AMCN$ este trapez;

$\mathscr{A}_{AMCN} = \dfrac{504+300\sqrt{3}}{175}$ cm^2.



34. Fie $DD' \perp AB$, $D' \in (AB)$. Notăm $AB = a$; $AD = b$; $AD' = x \Rightarrow D'B = a - x$.

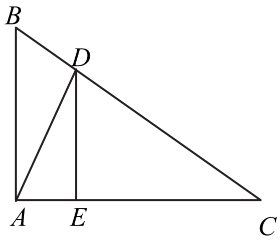
În $\Delta DD'B$ cu m($\sphericalangle DD'B$) = 90° $\overset{\text{T. Pitagora}}{\Rightarrow}$ $DB^2 = b^2 + (a - x)^2$ (1).

În $\Delta AD'D$ cu m($\sphericalangle AD'D$) = 90° $\overset{\text{T. Pitagora}}{\Rightarrow}$ $b^2 = h^2 - x^2$ (2).

Din (1) şi (2) $\Rightarrow DB^2 = a^2 + b^2 - 2ax$ (3).

Fie $AA' \perp CD \Rightarrow A'D = x \Rightarrow A'C = a - x$. În $\Delta AA'C$ cu m($\sphericalangle AA'C$) = 90° $\overset{\text{T. Pitagora}}{\Rightarrow}$ $AC^2 = AA'^2 + A'C^2 \Rightarrow AC^2 = (a + x)^2 + h^2 \Rightarrow AC^2 = a^2 + 20x + x^2 + b^2 - x^2 \Rightarrow AC^2 = a^2 + b^2 + 2ax$ (4). Din (3) şi (4) $\Rightarrow BD^2 + AC^2 = 2(a^2 + b^2)$.

4.5. Probleme pentru olimpiade şi concursuri



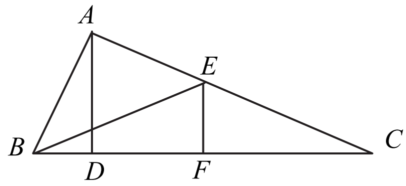
1. a) $AB^2 \cdot DC = AC^2 \cdot DB \Leftrightarrow (AD^2 + BD^2) \cdot DC = (AD^2 + CD^2) \cdot BD \Leftrightarrow AD^2(DC - DB) + BD \cdot DC(BD - DC) = 0 \Leftrightarrow (AD^2 - BD \cdot DC)(DC - DB) = 0$. Dar $AB \neq AC \Rightarrow DC \neq DB$. Rămâne $AD^2 = BD \cdot DC \Leftrightarrow$ m($\sphericalangle A$) = 90°;

b) $\dfrac{1}{AD^2} = \dfrac{1}{AB^2} + \dfrac{1}{AC^2} \Leftrightarrow \dfrac{1}{AD^2} = \dfrac{1}{AD^2 + BD^2} + \dfrac{1}{AD^2 + DC^2} \Leftrightarrow AD^4 = BD^2 \cdot CD^2 \Leftrightarrow AD^2 = BD \cdot CD \Leftrightarrow$ m($\sphericalangle A$) = 90°;

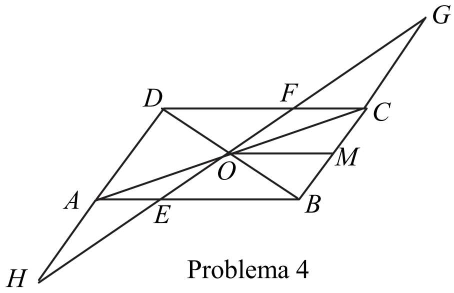
c) În ΔADE avem $AE \cdot AC = AD^2$. Cum $AE \cdot AC = BD \cdot CD$ sau $AD^2 = BD \cdot CD$.

2. Avem $ah_a = bh_b = ch_c$. Atunci $\dfrac{1}{h_a^2} = \dfrac{1}{h_b^2} + \dfrac{1}{h_c^2} \Leftrightarrow a^2 = b^2 + c^2$. **3.** Fie $AD \perp BC$, $D \in BC$. Fie $DF = FC = a$. Avem $FB^2 - FCD^2 = AB^2 \Leftrightarrow (BD + x)^2 - x^2 = AD^2 + BD^2 \Leftrightarrow BD \cdot DC = AB^2 \Leftrightarrow$

$\Leftrightarrow$ m($\measuredangle A$) = 90°. **4.** Fie $OM \parallel AB$, $M \in BC$. Avem $\Delta GEB \sim \Delta GOM$, deci $\dfrac{AB}{GH} = \dfrac{EB}{PE}$, $\dfrac{BC}{EF} = \dfrac{GB}{GE}$. Atunci avem: $BE^2 + BG^2 = PE^2 \Leftrightarrow AB \perp BC$.

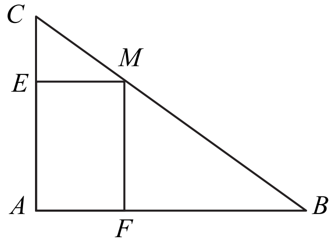


Problema 3

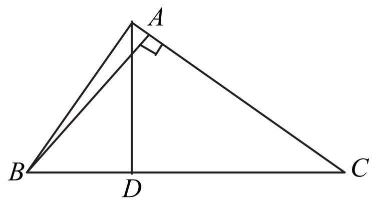


Problema 4

5. Fie $MF \parallel AC$, $MF \parallel AC$. Avem $\dfrac{AB}{ME} = \dfrac{BC}{MC}$, $\dfrac{AC}{AE} = \dfrac{BC}{MB}$, de unde $AB \cdot MC = BC \cdot ME$, $MC \cdot MB = BC \cdot AE$. Atunci $(AB \cdot MC)^2 + (AC \cdot MB)^2 = BC^2 \cdot MA^2 \Leftrightarrow BC^2 \cdot ME^2 + BC^2 \cdot AE^2 = BC^2 \cdot MA^2 \Leftrightarrow ME^2 + AE^2 = AM^2 \Leftrightarrow$ m($\measuredangle FAC$) = 90°. **6.** a) Fie m($\measuredangle C$) < 90°. Avem $AB^2 - BD^2 = AD^2 = AC^2 - DC^2 \Rightarrow AB^2 = AC^2 + BD^2 - CD^2 = AC^2 + (BD + CD)(BD - CD) = AC^2 + BC(BC - 2DC) = AC^2 + BC^2 - CB \cdot CD$; b) Înlocuim $BD - CD = BC$, $BD + CD = BC + 2CD$.



Problema 5

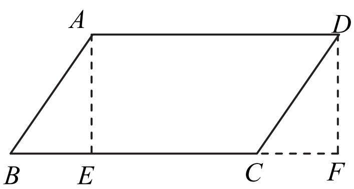


Problema 6

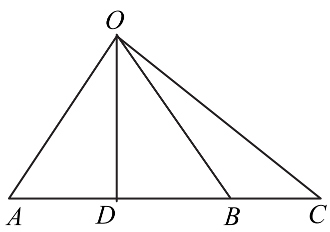
7. Luăm cazul de la **6** a). Avem $AD^2 = AC^2 - CD^2 = b^2 - \dfrac{(a^2 + b^2 - c^2)^2}{4a^2} =$

$= \dfrac{(2ab)^2 - (a^2 + b^2 - c^2)}{4a^2} = \dfrac{1}{4a^2}\left(2ab + a^2 + b^2 - c^2\right)\left(2ab - a^2 - b^2 + c^2\right) =$

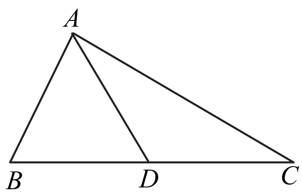
$= \dfrac{1}{4a^2}\left(a + b + c\right)(a + b + c)(c + a - b)(c - a + b)$. Cum $a + b - c = (2p - c) - c = 2(p - c)$ şi analoagele, avem: $AD = \dfrac{2}{a}\sqrt{p(p-a)(p-b)(p-c)}$. **8.** Fie $AC \perp BC$, $DF \perp BC$, $E \in BC$, $F \in BC$. $AC^2 + BD^2 = (AB^2 + BC^2 - 2BC \cdot BE) + BC^2 + CD^2 + 2BC \cdot CF = AB^2 + BC^2 + AD^2 + CD^2$.



Problema 8



Problema 9



Problema 10

9. Fie $OD \perp BC$. Luăm cazul $D \in (AB)$. $OA^2 = AB^2 + OB^2 - 2AB \cdot DB \mid BC$ (1); $OC^2 = OB^2 + BC^2 + 2BC \cdot DB \mid AB$ (2). Din (1), (2) $\Rightarrow OA^2 \cdot BC + OC^2 \cdot AB = AB^2 \cdot BC + OB^2(BC + AB) + BC^2 \cdot AB = AB \cdot BC(BC + AB) + OB^2 \cdot AC \Rightarrow OA^2 \cdot BC - OB^2 \cdot AC + DC^2 \cdot AB = AB \cdot BC \cdot CA$.

10. $AB^2 \cdot MC - AM^2 \cdot BC + AC^2 \cdot BM = BM \cdot MC \cdot BC \Leftrightarrow c^2 \cdot \frac{a}{2} - m^2 a + b^2 \frac{a}{2} = \frac{a}{2} \cdot \frac{a}{2} \cdot a \Leftrightarrow$ $\Leftrightarrow 4m_a^{\ 2} = 2(b^2 + c^2) - a^2$. **11.** Avem $AB^2 \cdot CD - AD^2 \cdot BC + AC^2 \cdot BD = BD \cdot CD \cdot BC \Leftrightarrow$

$$\Leftrightarrow \frac{c^2 ab}{b+c} - al_a^2 + \frac{b^2 ac}{b+c} = \frac{ab \cdot ac \cdot a}{(b+c)^2} \Rightarrow l_a^2 = \frac{bc\left[(b+c)^2 - a^2\right]}{(b+c)^2} \Rightarrow l_a = \frac{2}{b+c}\sqrt{bcp(p-a)}\ .$$

12. a) $m_a \geq m_b \Leftrightarrow 4m_a^2 \geq 4m_b^2 \Leftrightarrow 2\left(b^2 + c^2\right) - a^2 \geq 2\left(b^2 + c^2\right) - b^2 \Leftrightarrow b^2 \geq a^2 \Leftrightarrow b \geq a$;

b) $l_a^2 \leq l_b^2 \Leftrightarrow \frac{4bcp(p-a)}{(b+c)^2} \leq \frac{4acp(p-b)}{(a+c)^2} \Leftrightarrow b(b+c-a)(a+c)^2 \leq a(a+c-b)(b+c)^2 \Leftrightarrow a \leq b$;

c) $4\sum m_a^2 = \sum\left(2\left(b^2 + c^2\right) - a^2\right) = 3\sum a^2$.

TESTE DE EVALUARE

Testul 1

I. 1. $4\sqrt{3}$ cm. **2.** $3\sqrt{6}$ cm. **3.** $3\sqrt{5}$ cm. **4.** 12 cm. **5.** $4\sqrt{3}$ cm. **6.** 4 cm.

II 1. a) $12\sqrt{3}$ cm şi 12 cm; b) $144\sqrt{3}$ cm^2; **2.** a) $AB = 12$ cm; b) $\mathscr{P}_{\Delta ABC} = 12\left(3 + \sqrt{3}\right)$ cm.

III 1. a) $P = 54$ cm; b) $AC = 6\sqrt{13}$ cm; c) d(O, AB) = $\frac{12\sqrt{3}}{7}$ cm; d(O, CD) = $\frac{9\sqrt{3}}{7}$ cm.

Testul 2

I. 1. 9 cm. **2.** 18 cm. **3.** $3\sqrt{5}$ cm. **4.** $6\sqrt{2}$ cm. **5.** $4\sqrt{3}$ cm. **6.** $8\sqrt{2}$ cm.

II. 1. a) $AB = 6\sqrt{3}$ cm; b) 60°. **2.** a) 24 cm; b) $24\left(\sqrt{3} + 1\right)$ cm.

III 1. a) $\mathscr{A}_{ABCD} = 108\sqrt{3}$ cm^2; b) $AC = 12\sqrt{3}$ cm; c) ΔMAB este echilateral. $\mathscr{A}_{\Delta MAB} = 144\sqrt{3}$ cm^2.

Testul 3

I. 1. 6 cm. **2.** $2\sqrt{3}$ cm. **3.** $8\sqrt{3}$ cm. **4.** 36 cm. **5.** $2\sqrt{3}$ cm. **6.** 25 cm.

II. 1. a) 24 cm; b) $18\sqrt{3}$ cm. **2.** a) $AC = 16$ cm; $\mathscr{P}_{\Delta ABC} = 48$ cm; b) $h = \frac{c_1 \cdot c_2}{c_p} = \frac{12 \cdot 16}{20} = 9{,}6$ cm.

III. a) $BC = 4\sqrt{7}$ cm; b) $AC = 4\sqrt{10}$ cm; $BD = 4\sqrt{15}$ cm; c) $\Delta COD \sim \Delta AOB \overset{\text{T.F.A.}}{\Rightarrow} \frac{CO}{OA} = \frac{OD}{OB} =$

$= \frac{CD}{AB} = \frac{2}{3} \Rightarrow \frac{CO}{AC} = \frac{2}{5} \Rightarrow CO = \frac{2}{5} \cdot 4\sqrt{10} = \frac{8\sqrt{10}}{5}$ cm $\Rightarrow AO = 4\sqrt{10} - \frac{8\sqrt{10}}{5} = \frac{12\sqrt{10}}{5}$ cm;

$\frac{OD}{OB} = \frac{2}{3} \Rightarrow \frac{OD}{BD} = \frac{2}{5} \Rightarrow OD = \frac{2}{5} \cdot 4\sqrt{15} = \frac{8\sqrt{15}}{5}$ cm $\Rightarrow OB = \frac{12\sqrt{15}}{5}$ cm; $\mathscr{A}_{\Delta COB} = \frac{c_1 \cdot c_2}{2} =$

$$= \frac{OC \cdot OB}{2} = \frac{\frac{8\sqrt{10}}{5} \cdot \frac{12\sqrt{15}}{5}}{2} = \frac{96 \cdot \sqrt{150}}{50} = \frac{\overset{48}{\cancel{96}} \cdot \cancel{5}\sqrt{6}}{\cancel{50}_{\cancel{10}_5}} = \frac{48 \cdot \sqrt{6}}{5} \text{ cm}^2;$$

$$\mathscr{A}_{\Delta BOC} = \frac{BC \cdot \mathrm{d}(O,\, BC)}{24} \Rightarrow \frac{48\sqrt{6}}{5} = \frac{4\sqrt{7} \cdot \mathrm{d}(O,\, BC)}{2} \Rightarrow \mathrm{d}(O,\, BC) = \frac{2 \cdot \frac{\overset{24}{\cancel{48}}\sqrt{6}}{5}}{\cancel{4}\sqrt{7}} = \frac{24\sqrt{6}}{5\sqrt{7}} =$$

$= \frac{24 \cdot \sqrt{42}}{35}$ cm. Sau d(O, BC) $= \frac{c_1 - c_2}{c_p} = \frac{OC \cdot OB}{BC} = \frac{\frac{8\sqrt{10}}{5} \cdot \frac{12\sqrt{15}}{5}}{4\sqrt{7}} = \frac{\frac{96\sqrt{150}}{25}}{4\sqrt{7}} = \frac{\overset{24}{\cancel{96}} \cdot \cancel{5}\sqrt{6}}{\cancel{25}_5 \cdot \cancel{4}\sqrt{7}} =$

$= \frac{24\sqrt{6}}{5\sqrt{7}} = \frac{24\sqrt{42}}{35}$ cm.

Testul 4

I. 1. 15 cm. **2.** $6\sqrt{3}$ cm. **3.** $4\sqrt{3}$ cm. **4.** 10 cm. **5.** $2\sqrt{3}$ cm. **6.** 15 cm.

II. 1. a) $AC = 12$ cm; $BD = 12\sqrt{3}$ cm; b) $d(O, AB) = 3\sqrt{3}$ cm. **2.** a) $18\left(\sqrt{3}+1\right)$ cm; b) 9 cm;

III. a) $\mathscr{A}_{ABCD} = 90\sqrt{3}$ cm^2; b) $AC = 6\sqrt{7}$ cm; $BD = 12\sqrt{3}$ cm; c) ΔMAB este triunghi dreptunghic cu m($\sphericalangle AMB$) = 30°; $AB = 18$ cm $\Rightarrow MB = 36$ cm $\Rightarrow AM = 18\sqrt{3}$ cm $\Rightarrow \mathscr{P}_{\Delta AMB} = 18\left(3+\sqrt{3}\right)$ cm.

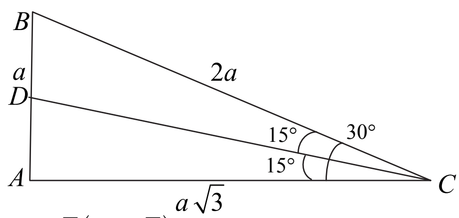
CAPITOLUL 5
Noţiuni de trigonometrie. Arii

5.1. Noţiuni de trigonometrie în triunghiul dreptunghic; sinusul, cosinusul, tangenta şi cotangenta unui unghi ascuţit

1. a) $BC = 13$ cm; $\sin B = \frac{5}{13}$; $\cos B = \frac{12}{13}$; $\operatorname{tg} B = \frac{5}{12}$; $\operatorname{ctg} B = \frac{12}{5}$; $\sin C = \frac{12}{13}$; $\cos C = \frac{5}{13}$; $\operatorname{tg} C = \frac{12}{5}$; $\operatorname{ctg} C = \frac{5}{12}$; b) $AC = 24$ cm; $\cos B = \frac{5}{13}$; $\operatorname{tg} B = \frac{12}{5}$; $\operatorname{ctg} C = \frac{12}{5}$; c) $AB = 12$ cm; $\cos B = \frac{3}{5}$; $\cos C = \frac{4}{5}$; $\sin B = \frac{4}{5}$; $\sin C = \frac{3}{5}$; $\operatorname{tg} B = \frac{4}{3}$; $\operatorname{ctg} B = \frac{3}{4}$; $\operatorname{tg} C = \frac{3}{4}$; $\operatorname{ctg} C = \frac{4}{3}$; d) $BC = 10$ cm; $AC = 6$ cm; $\cos B = \frac{4}{5}$; $\operatorname{tg} C = \frac{4}{3}$; $\operatorname{ctg} B = \frac{4}{3}$; e) $AB = 24$ cm, $BC = 30$ cm; $\sin B = \frac{3}{5}$; $\cos C = \frac{3}{5}$; $\cos B = \frac{4}{5}$; $\sin C = \frac{4}{5}$; f) $BC = 25$ cm; $AC = 24$ cm; $\sin B = \frac{24}{25}$; $\sin C = \frac{7}{25}$; $\cos C = \frac{24}{25}$; $\operatorname{tg} B = \frac{24}{7}$; $\operatorname{ctg} C = \frac{24}{7}$. **2.** $BC = 8\sqrt{3}$ cm; $AB = 4\sqrt{3}$ cm; $\sin C = \frac{1}{2}$; $\cos B = \frac{1}{2}$, $\operatorname{ctg} C = \frac{\sqrt{3}}{3}$. **3.** $BC = 16\sqrt{3}$ cm şi $B - b = 8\sqrt{3}$ cm. **4.** a) $\frac{\sqrt{6}-\sqrt{2}}{4}$; b) $\frac{\sqrt{6}+\sqrt{2}}{4}$;

c) $\frac{\sqrt{2}+\sqrt{6}}{4}$; d) $\frac{\sqrt{6}-\sqrt{2}}{4}$; e) $\frac{\sqrt{2}-\sqrt{6}}{4}$; f) $\frac{\sqrt{6}+\sqrt{2}}{4}$; g) $\frac{\sqrt{3}}{2}$; h) $\frac{\sqrt{2}}{2}$; i) 0; j) 1; k) $\frac{\sqrt{3}}{3}$; l) $\sqrt{2}-1$; m) $2-\sqrt{3}$; n) $\frac{1}{2}$; o) $-\frac{1}{2}$. **5.** BC = 32 cm; m($\sphericalangle B$) = 60°; m($\sphericalangle C$) = 30°. **6.** AB = 24 cm; m($\sphericalangle ABC$) = 60°, m($\sphericalangle BAD$) = 120°. **7.** AB = 24 cm; sin($\sphericalangle BAC$) = $\frac{5}{13}$; tg($\sphericalangle ACD$) = $\frac{12}{5}$; cos($\sphericalangle ACB$) = $\frac{5}{13}$; ctg($\sphericalangle ACD$) = $\frac{12}{5}$; sin^2($\sphericalangle BAC$) + cos^2($\sphericalangle BAC$) $=\left(\frac{5}{13}\right)^2+\left(\frac{12}{13}\right)^2=1$; sin($\sphericalangle BAC$) = cos($\sphericalangle ACB$) $=\frac{5}{13}$. **8.** Se construieşte $AD \perp BC$; AD = 16 cm; sin $B=\frac{4}{5}$; cos $C=\frac{3}{5}$. **9.** AC = 18 cm, $BC=12\sqrt{3}$ cm; AD = 9 cm; $BD=3\sqrt{3}$ cm; $CD=9\sqrt{3}$ cm. **10.** $AC=4\sqrt{3}$ cm; $BC=8\sqrt{3}$ cm; AD = 6 cm; $CD=2\sqrt{3}$ cm; $BD=6\sqrt{3}$ cm. **11.** AB = 24 cm; AC = 10 cm; $\mathscr{P}_{\Delta ABC}$ = 60 cm; $\mathscr{A}_{\Delta ABC}$ = 120 cm^2. **12.** $AB=10\sqrt{3}$ cm; $AC=24\sqrt{3}$ cm; $AD=\frac{120\sqrt{3}}{13}$ cm; $BD=$ $=\frac{50\sqrt{3}}{13}$ cm; $CD=\frac{288\sqrt{3}}{13}$ cm; cos $\sphericalangle B=\frac{5}{13}$; cos $\sphericalangle C=\frac{12}{13}$; sin $\sphericalangle C=\frac{5}{13}$; tg $\sphericalangle C=\frac{12}{5}=$ = ctg $\sphericalangle B$; ctg $\sphericalangle C=\frac{5}{12}$. **13.** a) $BC=12\sqrt{2}$ cm; $BD=12\sqrt{2}$ cm; $\mathscr{P}_{ABCD}=12\left(4+\sqrt{2}\right)$ cm; $\mathscr{A}_{ABCD}=$ = 216 cm^2; b) m($\sphericalangle BDC$) = m($\sphericalangle BCD$) = 45° $\Rightarrow$ m($\sphericalangle DBC$) = 90° $\Rightarrow$ $BD \perp BC$. **14.** AB = 12 cm; $BC=6+6\sqrt{3}$ cm; $AC=6\sqrt{6}$ cm; $\mathscr{P}_{ABCD}=6\left(3+\sqrt{3}+\sqrt{6}\right)$ cm; $\mathscr{A}_{\Delta ABC}=18\sqrt{3}\left(\sqrt{3}+1\right)$ cm^2. **15.** lățimea = 24 cm; lungimea = 32 cm; diagonala = 40 cm; sin($\sphericalangle CAB$) $=\frac{3}{5}$; sin($\sphericalangle ACD$) = $\frac{3}{5}$; cos($\sphericalangle$CAD) = $\frac{3}{5}$; ctg($\sphericalangle BAC$) = $\frac{4}{3}$; tg($\sphericalangle CAD$) = $\frac{4}{3}$. **16.** tg($\sphericalangle C$) $=\frac{AD}{DC}$ (unde $AD \perp BC$, $D \in$ $\in (BC)$) $\Rightarrow AD = 4k$; $CD = 3k \Rightarrow k=3\sqrt{7} \Rightarrow AD=12\sqrt{7}$ cm; $DC=9\sqrt{7}$ cm $\Rightarrow$ cos($\sphericalangle C$) $=\frac{3}{5}$; sin($\sphericalangle C$) $=\frac{4}{5}$. **17.** a) $20\left(2+\sqrt{2}\right)$ cm; $\mathscr{A}_{ABCD}$ = 200 cm^2; b) $AC=BD=10\sqrt{5}$ cm; sin($\sphericalangle CAB$) = $=\frac{20}{10\sqrt{5}}=\frac{2\sqrt{5}}{5}$; tg ($\sphericalangle CAB$) = $\frac{10}{20}=\frac{1}{2}$. **18.** m($\sphericalangle ABC$) = 60°; m($\sphericalangle BAD$) = 120°; AC = 24 cm; $BD=24\sqrt{3}$ cm, d(O, AB) $=6\sqrt{3}$ cm. **19.** Fie $AD \perp BC$, $D \in (BC)$. Notăm $BD = x \Rightarrow CD = 14 - x$. Din aplicarea teoremei lui Pitagora în Δ dreptunghice ADB, respectiv $ADC \Rightarrow 169 - x^2 =$ $= 225 - (14 - x)^2 \Leftrightarrow x = 5 \Rightarrow BD$ = 5 cm $\Rightarrow CD$ = 9 cm $\Rightarrow AD$ = 12 cm, sin $\sphericalangle B=\frac{12}{13}$; cos $\sphericalangle B=$ $=\frac{5}{13}$; tg $\sphericalangle B=\frac{12}{5}$; ctg $\sphericalangle B=\frac{5}{12}$; sin $\sphericalangle C=\frac{4}{5}$; cos $\sphericalangle C=\frac{3}{5}$; tg $\sphericalangle C=\frac{4}{3}$; ctg $\sphericalangle C=\frac{3}{4}$.

20. Fie triunghiul ΔABC cu m($\sphericalangle B$) = 90°, m($\sphericalangle C$) = 30°; $AB = a \Rightarrow AC = a\sqrt{3} \Rightarrow BC = 2a$ şi (CD, $D \in$ (AB) bisectoarea unghiului $\sphericalangle ACB$. Din teorema bisectoarei $\Rightarrow \dfrac{AD}{DB} = \dfrac{AB}{BC} \Rightarrow$



$\Rightarrow \dfrac{AD}{a - AD} = \dfrac{\cancel{a}\sqrt{3}}{2\cancel{a}} \Leftrightarrow 2AD = a\sqrt{3} - AD\sqrt{3} \Leftrightarrow AD = a\sqrt{3}\left(2 - \sqrt{3}\right)$. În ΔCAD cu m($\sphericalangle CAD$) = $= 90° \overset{\text{T. Pitagora}}{\Rightarrow} CD^2 = AC^2 + AD^2 \Rightarrow CD^2 = 3a^2 + 3a^2\left(2 - \sqrt{3}\right)^2 = 3a^2\left(1 + 4 - 4\sqrt{3} + 3\right) =$

$= 3a^2\left(8 - 4\sqrt{3}\right) = 12a^2\left(2 - \sqrt{3}\right) \Rightarrow CD = 2a\sqrt{3}\left(\sqrt{6} - \sqrt{2}\right)$. În ΔCAD cu m($\sphericalangle CAD$) = 90° şi

m($\sphericalangle ACD$) = 15° $\Rightarrow$ a) $\sin 15° = \dfrac{AD}{CD} = \dfrac{\cancel{a\sqrt{3}}\left(2 - \sqrt{3}\right)}{\cancel{a\sqrt{3}}\left(\sqrt{6} - \sqrt{2}\right)} = \dfrac{\left(2 - \sqrt{3}\right)\left(\sqrt{6} + \sqrt{2}\right)}{4} =$

$= \dfrac{2\sqrt{6} + 2\sqrt{2} - 3\sqrt{2} - \sqrt{6}}{4} = \dfrac{\sqrt{6} - \sqrt{2}}{4}$; b) $\cos 15° = \dfrac{AC}{CD} = \dfrac{\cancel{a\sqrt{3}}}{\cancel{a\sqrt{3}}\left(\sqrt{6} - \sqrt{2}\right)} = \dfrac{\sqrt{6} + \sqrt{2}}{4}$;

c) $\text{tg}15° = \dfrac{\sin 15^\circ}{\cos 15^\circ} = \dfrac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} = \dfrac{\left(\sqrt{6} - \sqrt{2}\right)^2}{4} = 2 - \sqrt{3}$; d) $\text{ctg } 15° = \dfrac{1}{tg15^\circ} = 2 + \sqrt{3}$; e) $\sin 75° =$ $= \cos(90° - 75°) = \cos 15° = \dfrac{\sqrt{6} + \sqrt{2}}{4}$; f) $\cos 75° = \sin (90° - 75°) = \sin 15° = \dfrac{\sqrt{6} - \sqrt{2}}{4}$;

g) tg 75° = ctg (90° – 75°) = ctg 15° = $2 + \sqrt{3}$; h) ctg 75° = tg(90° – 75°) = tg 15° = $2 - \sqrt{3}$.

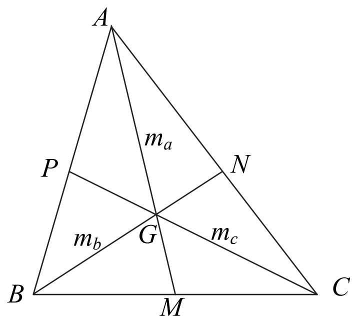
21. a) Fie M mijlocul lui BC . Se aplică teorema cosinusului.

a) În $\Delta AMB \overset{\text{T. cos.}}{\Rightarrow} AB^2 = AM^2 + BM^2 - 2 \cdot AM \cdot BM$;

$\cos(\sphericalangle AMB) \Leftrightarrow c^2 = m_a{}^2 + \dfrac{a^2}{4} - \cancel{2} \cdot m_a \cdot \dfrac{a}{\cancel{2}}\cos(\sphericalangle AMB) \Leftrightarrow$

$\Leftrightarrow c^2 = m_a{}^2 + \dfrac{a^2}{4} - a \cdot m_a \cdot \cos\left(\sphericalangle AMB\right)$ (1)

În $\Delta AMC \overset{\text{T. cos.}}{\Rightarrow} AC^2 = AM^2 + CM^2 + 2 \cdot AM \cdot MC \cdot \cos(180° -$ $- \sphericalangle AMC) \Leftrightarrow b^2\ m_a{}^2 + \dfrac{a^2}{4} + 2 \cdot m_a \cos(\sphericalangle AMB)$ (2).



Din (1) şi (2) $\Rightarrow b^2 + c^2 = 2m_a{}^2 + \dfrac{a^2}{2} \Leftrightarrow m_a^2 = \dfrac{2(b^2 + c^2) - a^2}{4}$.

Analog $m_b{}^2 = \dfrac{2(c^2 + a^2) - b^2}{4}$; $m_c{}^2 = \dfrac{2(a^2 + b^2) - c^2}{4}$;

b) $m_a{}^2 + m_b{}^2 + m_c{}^2 = \dfrac{2(b^2 + c^2) - a^2 + 2(c^2 + a^2) - b^2 + 2(a^2 + b^2) - c^2}{4} = \dfrac{3(a^2 + b^2 + c^2)}{4}$;

c) Din b) obţinem $\dfrac{3(a^2 + b^2 + c^2)}{4} = \dfrac{3}{2}\left(b^2 + c^2\right) \Leftrightarrow a^2 = b^2 + c^2 \overset{\text{R.T.Pitagora}}{\Rightarrow}$ m($\sphericalangle BAC$) = 90°;

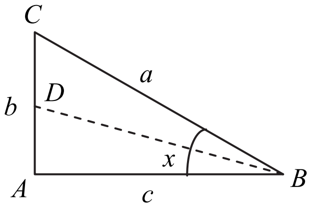
d) $m_a^2 = \dfrac{2(144+64)-100}{4} = 79 \Rightarrow m_a = \sqrt{79}$ cm; $m_b^2 = \dfrac{2(100+64)-144}{4} = 46 \Rightarrow m_b = \sqrt{46}$ cm;

$m_c^2 = \dfrac{2(100+144)-64}{4} = 106 \Rightarrow m_c = \sqrt{106}$ cm. **22.** Fie x = m($\sphericalangle B$) în ΔABC, unde m($\sphericalangle A$) = 90°.

a) $1+\mathrm{tg}^2 x = 1+\dfrac{\sin^2 x}{\cos^2 x} = \dfrac{\sin^2 x+\cos^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x}$; c) Fie t = tg x. Avem $\dfrac{1+t^4}{t^2+\dfrac{1}{t^2}} = t^2$;

d) $\dfrac{1+t}{1-t} + \dfrac{1+\dfrac{1}{t}}{1-\dfrac{1}{t}} = \dfrac{1+t}{1-t} + \dfrac{t+1}{t-1} = 0$.

23. a) Fie (BD bisectoarea unghiului B. Avem $\dfrac{AD}{DC} = \dfrac{AB}{DC} \Rightarrow$



$\Rightarrow AD = \dfrac{bc}{a+c} \Rightarrow \sin\dfrac{B}{2} = \dfrac{AD}{BD} = \dfrac{AD}{\sqrt{AB^2+AD^2}} = \dfrac{\dfrac{bc}{a+c}}{\sqrt{c^2+\dfrac{b^2c^2}{(a+c)^2}}} =$

$= \dfrac{b}{\sqrt{(a+c)^2+b^2}} = \sqrt{\dfrac{b^2}{a^2+b^2+2ac+b}} = \sqrt{\dfrac{a^2-c^2}{2a(a+c)}} = \sqrt{\dfrac{a-c}{2a}} = \sqrt{\dfrac{1-\dfrac{c}{a}}{2}} = \sqrt{\dfrac{1-\cos B}{2}}$;

b) Analog $\cos\dfrac{B}{2} = \dfrac{AB}{AD} = \sqrt{\dfrac{a+c}{2a}} = \sqrt{\dfrac{1+\cos B}{2}}$; c) $tg\dfrac{B}{2} = \sqrt{\dfrac{1-\cos B}{2}} : \sqrt{\dfrac{1+\cos B}{2}} = \sqrt{\dfrac{1-\cos B}{1+\cos B}}$.

24. a) $\dfrac{b^2}{\mathrm{tg}\, B} + \dfrac{c^2}{\mathrm{tg}\, C} = \dfrac{b^2c}{bc} + \dfrac{c^2b}{c} = 2bc = 4S$; b) $\dfrac{\sin B+\cos C}{\sin C+\cos B} = \dfrac{2\dfrac{b}{a}}{2\dfrac{c}{a}} = \dfrac{b}{c} = c\ \mathrm{tg}\, C$;

c) $\sqrt{\dfrac{a^2}{b^2}+\dfrac{a^2}{c^2}} = \sqrt{\dfrac{a^2(b^2+c^2)}{b^2c^2}} = \dfrac{a^2}{bc}$; d) $\left(1+\dfrac{b}{a}\right)\left(1+\dfrac{c}{a}\right) = \dfrac{(a+b)(a+c)}{a^2} = \dfrac{a^2+ab+ac+bc}{a^2} =$

$= \dfrac{a^2+b^2+c^2+2ab+2ac+2bc}{2a^2} = \dfrac{(a+b+c)^2}{2a^2} = \dfrac{2p^2}{a^2}$.

5.2. Rezolvarea triunghiului dreptunghic

1. BD = 18 cm; CD = 32 cm; BC = 50 cm; AC = 40 cm; sin B = cos $C = \dfrac{4}{5}$. **2.** BC = 50 cm; AD = 24 cm; AB = 30 cm; AC = 40 cm; sin B = cos $C = \dfrac{4}{5}$; tg $B = \dfrac{4}{3}$; ctg $B = \dfrac{3}{4}$; tg $B \cdot$ ctg B = $= \dfrac{4}{3}\cdot\dfrac{3}{4} = 1$. **3.** a) $\mathscr{P}_{\Delta ABC} = 12\left(3+\sqrt{3}\right)$ cm; b) $h = 6\sqrt{3}$ cm: c) sin B = cos $C = \dfrac{\sqrt{3}}{2}$; cos B = sin C =

$=\dfrac{1}{2}$. **4.** $CD=16\sqrt{3}$ cm; $BC=25\sqrt{3}$; $AB=15\sqrt{3}$ cm; $AC=20\sqrt{3}$ cm; sin $B=\dfrac{4}{5}$; sin $C=\dfrac{3}{5}$.
5. BC = 60 cm; BD = 38,4 cm; AD = 28,8 cm; AB = 48 cm; sin $B=\dfrac{3}{5}$; tg $C=\dfrac{4}{3}$. **6.** BC = 26 cm; AC = 10 cm; $AD=\dfrac{120}{13}$ cm; $BD=\dfrac{288}{13}$ cm; $CD=\dfrac{50}{13}$ cm; sin B = cos $C=\dfrac{5}{13}$. **7.** $AC=36\sqrt{3}$ cm; AB = 36 cm; BC = 72 cm; DB = 18 cm; DC = 54 cm. Fie M mijlocul lui (AC) $\Rightarrow$ BM = $=18\sqrt{7}$ cm. **8.** $AB=18\sqrt{3}$ cm; AD = 27 cm; $DC=27\sqrt{3}$ cm; $BC=36\sqrt{3}$ cm; AC = 54 cm. **9.** $AC=24\sqrt{3}$ cm; $BC=48\sqrt{3}$ cm; AB = 72 cm; AD = 36 cm; $BD=36\sqrt{3}$ cm. **10.** AB = 12 cm; $AC=12\sqrt{3}$ cm; AD = 24 cm; BD = 12 cm; CD = 32 cm; tg C = tg $30°=\dfrac{\sqrt{3}}{3}$. **11.** AB = 30 cm; AC = 40 cm; $AD=6\sqrt{3}$ cm; BD = 6 cm; CD = 18 cm; sin $B=\dfrac{4}{5}$. **12.** $\mathcal{P}_{ABCD}=12\left(\sqrt{3}+1\right)$ cm şi d$(B, AC)=3\sqrt{3}$ cm. **13.** Notăm $BD=x \Rightarrow 256=x(x+7,2) \Rightarrow x^2+7,2x-256=0 \Rightarrow (x+3,6)^2-$ $-16,4^2=0 \Leftrightarrow (x+20)(x-12,8)=0 \Rightarrow x=12,8$ cm $\Rightarrow BD=12,8$ cm $\Rightarrow BC=20$ cm $\Rightarrow AC=$ $=12$ cm; sin B = cos $C=\dfrac{3}{5}$; tg B = tg $C=\dfrac{3}{4}$. **14.** a) $AB=6\sqrt{3}$ cm; $AC=3\sqrt{6}$ cm; $BC=$ $=3\left(3+\sqrt{3}\right)$ cm; b) $BE=\dfrac{3\left(3\sqrt{2}+\sqrt{6}\right)}{2}$ cm; c) În ΔABE cu m($\sphericalangle AEB$) = 90° şi m($\sphericalangle EBA$) = 15°;

$$\cos 15° = \frac{BE}{AB}=\frac{\frac{3\left(3\sqrt{2}+\sqrt{6}\right)}{2}}{6\sqrt{3}}=\frac{3\sqrt{2}+\sqrt{6}}{4\sqrt{3}}=\frac{3\sqrt{6}+3\sqrt{2}}{4\cdot 3}=\frac{\sqrt{6}+\sqrt{2}}{4}.$$

Pentru că ΔACE este un triunghi dreptunghic isoscel $\Rightarrow$ $AE=BE-AC=\dfrac{3\left(3\sqrt{2}+\sqrt{6}\right)}{2}-3\sqrt{6}=\dfrac{9\sqrt{2}+3\sqrt{6}-6\sqrt{6}}{2}=$

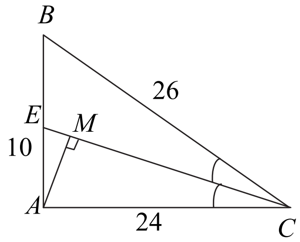
$$=\frac{9\sqrt{2}-3\sqrt{6}}{2}=\frac{3\left(3\sqrt{2}-\sqrt{6}\right)}{2} \text{ şi } \sin 15°=\frac{AE}{AB}=\frac{\frac{3\left(3\sqrt{2}-\sqrt{6}\right)}{2}}{6\sqrt{3}}=\frac{3\sqrt{2}-\sqrt{6}}{4\sqrt{3}}=\frac{3\sqrt{6}-3\sqrt{2}}{4\cdot 3}=$$

$=\dfrac{\sqrt{6}-\sqrt{2}}{4}$. **15.** a) $\begin{cases} B+b=48 \\ B-b=20 \end{cases} \Rightarrow \begin{cases} B=34 \text{ cm} \\ b=14 \text{ cm} \end{cases}$; $b=\dfrac{B-b}{2}=10$ cm. $AD=BC=10\sqrt{2}$ cm $\Rightarrow$ $\Rightarrow \mathcal{P}_{ABCD}=4\left(5\sqrt{2}+12\right)$ cu $\mathcal{A}_{ABCD}$ = 240 cm^2; b) $AC=BD$ = 26 cm; c) Fie $AE \perp BC \Rightarrow$ $\Rightarrow \Delta AEB$ este triunghi dreptunghic isoscel şi avem sin B = sin $45°=\dfrac{AF}{AB}\Rightarrow\dfrac{\sqrt{2}}{2}=\dfrac{AE}{34}\Rightarrow AE=$ $17\sqrt{2}$ cm $\Rightarrow$ d$(A, BC)=17\sqrt{2}$ cm.

5.3. Aria triunghiului

1. a) 18 cm^2; b) 24 cm^2: c) 54 cm^2; d) $6\sqrt{3}$ cm^2. **2.** a) $36\sqrt{3}$ cm^2; b) 30 cm^2; c) 32 cm^2. **3.** a) 8 cm; b) $12\sqrt{2}$ cm; c) 20 cm; d) $12\sqrt{3}$ cm. **4.** $\mathcal{A}_{\Delta ABC}$ = 96 cm^2, $\mathcal{P}_{\Delta ABC}$ = 48 cm. **5.** $\mathcal{P}_{\Delta ABC}$ = 112 cm,

$\mathcal{A}_{\Delta ABC} = 336\text{ cm}^2$. **6.** $\mathcal{P}_{\Delta ABC} = 18\left(\sqrt{3}+1\right)$ cm; $\mathcal{A}_{\Delta ABC} = 54\sqrt{3}\text{ cm}^2$. **7.** $\mathcal{A}_{\Delta ADB} = 160\text{ cm}^2 \Rightarrow \mathcal{A}_{\Delta BDC} = 160\text{ cm}^2 \Rightarrow d(D, BC) = 8$ cm. **8.** $\mathcal{P}_{\Delta ABC} = 6\sqrt{3}\left(3+\sqrt{5}\right)$ cm; $\mathcal{A}_{\Delta ABC} = 108\text{ cm}^2$. **9.** a) $AB = 4\sqrt{13}$ cm; $AC = 6\sqrt{13}$ cm; $BC = 26$ cm; $\mathcal{P}_{\Delta ABC} = 2\sqrt{13}\left(5+\sqrt{13}\right)$ cm; b) $AD = 12$ cm. **10.** $\mathcal{P}_{\Delta ABC} = 12\sqrt{3}\left(\sqrt{3}+1\right)$ cm; $\mathcal{A}_{\Delta ABC} = 72\sqrt{3}\text{ cm}^2$. **11.** a) $\mathcal{A}_{\Delta ABC} = 36\sqrt{3}\text{ cm}^2$; $\mathcal{P}_{\Delta ABC} = 12\left(2+\sqrt{3}\right)$ cm; b) $CE = 6\sqrt{3}$. **12.** $BC^2 = AB^2 + AC^2 \Rightarrow AB^2 + AC^2 = 2500$; $\mathcal{A}_{\Delta ABC} = \dfrac{AB \cdot AC}{2} \Rightarrow AB \cdot AC = 1200 \Rightarrow 2\, AB \cdot AC = 2400 \Rightarrow (AB + AC)^2 = 4900 \Rightarrow AB + AC = 70$ cm $\Rightarrow \mathcal{P}_{\Delta ABC} = AB + AC + BC = 70 + 50 = 120$ cm. **13.** a) $\mathcal{P}_{\Delta ABC} = 4\left(3+\sqrt{3}+\sqrt{6}\right)$ cm; $\mathcal{A}_{\Delta ABC} = 8\sqrt{3}\left(1+\sqrt{3}\right)\text{cm}^2$; b) Fie $BE \perp AC$. Se exprimă în două moduri $\mathcal{A}_{\Delta ABC} \Rightarrow BE = 2\left(\sqrt{6}+\sqrt{2}\right)$ cm; c) m($\measuredangle ABE$) = 15°; cos($\measuredangle ABE$) = $\dfrac{BE}{AB} = \dfrac{\sqrt{6}-\sqrt{2}}{4}$; $AE = 2\left(\sqrt{6}-\sqrt{2}\right)$ cm; sin($\measuredangle ABE$) $= \dfrac{\sqrt{6}-\sqrt{2}}{4}$; tg($\measuredangle ABE$) $= \dfrac{AE}{AB} = 2-\sqrt{3}$; ctg($\measuredangle ABE$) $= 2+\sqrt{3}$.



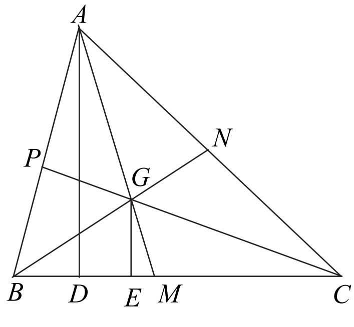
14. a) ΔABC este dreptunghic în A conform reciprocei teoremei lui Pitagora $\Rightarrow \mathcal{A}_{\Delta ABC} = 120\text{ cm}^2$; b) $AD = \dfrac{AB \cdot AC}{BC} = \dfrac{10 \cdot 24}{26} = \dfrac{120}{13}$ cm;

Conform teoremei bisectoarei: $\dfrac{AE}{EB} = \dfrac{AC}{BC} \Rightarrow \dfrac{AE}{EB} = \dfrac{24}{26} \Rightarrow$

$\Rightarrow \dfrac{AE}{AB} = \dfrac{24}{26} \Rightarrow \dfrac{AE}{10} = \dfrac{24}{50} \Rightarrow AE = \dfrac{24}{5}$ cm = 4,8 cm.

$\mathcal{A}_{\Delta ACE} = \dfrac{AC \cdot AE}{2} = \dfrac{24 \cdot 4{,}8}{2} = 24 \cdot 2{,}4 = 57{,}6\text{ cm}^2$. Conform teoremei lui Pitagora se obține: $CE = \dfrac{24}{5}\sqrt{26}$ cm. Fie $AM \perp CE$, $M \in CE$. Exprimăm aria ΔACE în două moduri și obținem:

$$57{,}6 = \frac{CE \cdot AM}{2} \Rightarrow \frac{576}{10} = \frac{\frac{24}{5} \cdot \sqrt{26} \cdot AM}{2} \Rightarrow AM = \frac{24}{\sqrt{26}} = \frac{24\sqrt{26}}{26} = \frac{12\sqrt{26}}{13}\text{ cm.}$$



15. a) Fie $AD \perp BC$ și $GE \perp BC \Rightarrow AD \parallel GE \overset{\text{T. F. A.}}{\Rightarrow}$

$\Rightarrow \Delta MGE \sim \Delta MAD \Rightarrow \dfrac{MG}{MA} = \dfrac{GE}{AD} = \dfrac{ME}{MD}$, dar

$\dfrac{MG}{MA} = \dfrac{1}{3} \Rightarrow \dfrac{GE}{AD} = \dfrac{1}{3} \Rightarrow GE = \dfrac{1}{3}AD$.

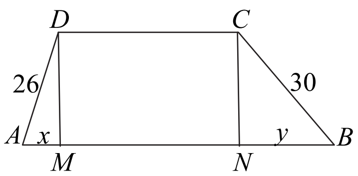
$\mathcal{A}_{\Delta BGC} = \dfrac{BC \cdot GE}{2} = \dfrac{1}{3} \cdot \dfrac{BC \cdot AD}{2} = \dfrac{1}{3} \cdot \mathcal{A}_{\Delta ABC}$;

b) Aplicăm formula lui Heron și obținem:
$\mathcal{A}_{\Delta BGC} = 336\text{ cm}^2 \Rightarrow \mathcal{A}_{\Delta ABC} = 1008\text{ cm}^2$.

5.4. Aria patrulaterelor

1. a) 144 cm^2; b) 27 cm^2; c) 240 cm^2; d) 140 cm^2; e) 40 cm^2. **2.** a) 64 cm^2; b) 24 cm^2; c) $25\sqrt{3}$ cm^2; **3.** a) 8 cm^2: b) 120 cm^2; **4.** a) 48 cm^2; b) 168 cm^2; c) 246 cm^2; d) 984 cm^2. **5.** a) $36\sqrt{2}$ cm^2; b) 36 cm^2. **6.** 52 cm. **7.** 44 cm. **8.** $\frac{63\sqrt{3}}{2}$ cm^2. **9.** a) $\mathscr{A}_{ABCD} = 540$ cm^2; b) $\mathscr{P}_{ABCD} = 12\left(\sqrt{34}+\sqrt{3}\right)$ cm; d(AB, CD) = $\frac{45\sqrt{34}}{17}$; c) d(AB, CD) = 22,5 cm. **10.** a) $\mathscr{A} = 3456$ cm^2; b) d(O, AB) = 28,8 cm. **11.** a) $200\sqrt{3}$ cm^2; b) 60°. **12.** a) 288 cm^2; b) $\frac{48\sqrt{5}}{5}$ cm. **13.** $\mathscr{A}_{ABCD} = 1512$ cm^2; d(AB, CD) = 33,6 cm; d(AD, BC) = $\frac{504}{13}$ cm. **14.** a) $\mathscr{P} = 16\left(1+\sqrt{3}\right)$ cm; b) $\mathscr{A} = 64\sqrt{3}$ cm^2. **15.** a) $\mathscr{A}_{ABCD} = 200$ cm^2; b) $\mathscr{P}_{ABCD} = 10 + 25 + 10\sqrt{2} + 15 = 50 + 10\sqrt{2} = 10\left(5+\sqrt{2}\right)$ cm; $AC = 5\sqrt{13}$ cm; $BD = 5\sqrt{29}$; c) $\mathscr{A}_{\Delta MDC} = 36\% \cdot \mathscr{A}_{\Delta MAB}$. **16.** $P_{ABCD} = 90$ cm; $\mathscr{A}_{ABCD} = 300$ cm^2.

17. Fie $DM \perp AB$; $CN \perp AB$, $M, N \in (AB)$. Notăm $AM = x$; $BN = y$. Avem relațiile $x + y = 60 - 32 \Rightarrow x + y = 28$. Aplicăm teorema lui Pitagora în ΔAMD şi în ΔBNC şi obținem: $26^2 - x^2 = 30^2 - y^2 \Leftrightarrow (y - x)(y + x) = 4 \cdot 56 \Leftrightarrow$ $\Leftrightarrow y - x = 8$ şi $y + x = 28 \Rightarrow y = 18$; $x = 10 \Rightarrow h = 24$ cm $\Rightarrow$

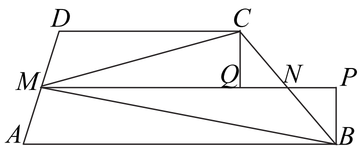


$\Rightarrow \mathscr{A}_{ABCD} = \frac{\left(60+32\right)\cdot 24}{2} = 1104\,cm^2$. **18.** $\mathscr{P}_{ABCD} = 60$ cm; $\mathscr{A}_{ABCD} = 108\sqrt{3}$ cm^2.

19. Fie N mijlocul lui BC şi $CQ \perp MN$, $BP \perp MN$;

$\mathscr{A}_{\Delta MBC} = \mathscr{A}_{\Delta MNB} + \mathscr{A}_{\Delta MNC} = \frac{MN \cdot BP}{2} + \frac{MN \cdot CQ}{2} =$

$= \frac{MN \cdot h}{2} = \frac{1}{2}\mathscr{A}_{ABCD} = 240$ cm^2.



20. a) m($\sphericalangle BAD$) = 45° $\Rightarrow$ ΔADB este triunghi dreptunghic isoscel $\Rightarrow$ $AD = DB = x$ şi $AB = x\sqrt{2}$. În ΔADO cu m($\sphericalangle ADO$) = 90° $\Rightarrow$ $AO^2 = AD^2 + DO^2 \Rightarrow \left(3\sqrt{5}\right)^2 = x^2 + \frac{x^2}{4} \Rightarrow 9 \cdot 5 = \frac{5x^2}{4} \Rightarrow x^2 = 36 \Rightarrow x = 6$ cm $\Rightarrow AD = DB = 6$ cm $\Rightarrow AB = 6\sqrt{2}$ cm; b) $\mathscr{A}_{ABCD} = AD = DB = 6$ cm $\Rightarrow AB = 6\sqrt{2}$ cm; b) $\mathscr{A}_{ABCD} = AD \cdot DB = 36$ cm^2; c) Fie $OM \perp AB \Rightarrow \Delta OMB$ este triunghi dreptunghic isoscel $\Rightarrow OM\sqrt{2} = BO \Rightarrow OM = \frac{3\sqrt{2}}{2}$ cm.

TESTE DE EVALUARE

Testul 1

I. 1. $AC = 16$ cm. **2.** $AB = 8\sqrt{3}$ cm. **3.** $\mathscr{P} = 30$ cm. **4.** $\mathscr{A} = 6$ cm^2. **5.** $\mathscr{A} = 27\sqrt{3}$ cm^2. **6.** cos C =

$= \dfrac{3\sqrt{10}}{10}$.

II. 1. a) $\mathscr{P} = 72$ cm; $\mathscr{A} = 162\sqrt{3}$ cm^2; b) d(A, CD) $= 9\sqrt{3}$ cm. **2.** a) $L = 32$ cm; $l = 24$ cm; b) sin ($\sphericalangle AOD$) $= \dfrac{24}{25}$.

III. a) $\mathscr{P}$= 66 cm; $\mathscr{A}$= 234 cm^2 ; b) $AC = 3\sqrt{41}$ cm; $BD = 12\sqrt{5}$ cm; c) d(D, BC) = 12 cm.

Testul 2

I. 1. 50 cm^2. **2.** 12 cm. **3.** 48 cm^2. **4.** $16\sqrt{2}$ cm. **5.** $\mathscr{A} = 36\sqrt{3}$ cm^2. **6.** 144 cm^2.

II. 1. a) $\mathscr{A} = 160\sqrt{5}$ cm^2; b) sin($\sphericalangle BAE$) $= \dfrac{4\sqrt{5}}{9}$. **2.** $\mathscr{A} = 100\sqrt{3}$ cm^2; $\mathscr{P}$= 60 cm.

III. a) $\Delta AMQ \equiv \Delta BNM \equiv \Delta CPN \equiv \Delta DQP$ (C.C.) $\Rightarrow [MQ] \equiv [MN] \equiv [PN] \equiv [QP] \Rightarrow MNPQ$ romb și m($\sphericalangle AMQ$) + m($\sphericalangle BMN$) = 90° $\Rightarrow$ m($\sphericalangle NMQ$) = 90° $\Rightarrow$ $MNPQ$ pătrat; b) $\mathscr{P}_{MNPQ} = 32\sqrt{10}$ cm; $\mathscr{A}_{MNPQ}$ = 640 cm^2; c) 576 cm^2.

Testul 3

I. 1. BC = 25 cm. **2.** $\mathscr{A}$= 216 cm^2. **3.** 336 cm^2. **4.** $48\sqrt{2}$ cm. **5.** 100 cm^2. **6.** A.

II. 1. a) 432 cm^2; b) $\dfrac{24}{25}$. **2.** 3600 cm^2.

III. a) BC = 24 cm; b) $CE = 8\sqrt{3}$ cm; c) 12 cm.

Testul 4

I. 1. 81 cm^2. **2.** 756 cm^2. **3.** $16\left(2+\sqrt{2}\right)$ cm. **4.** 60°. **5.** d $= 6\sqrt{13}$ cm. **6.** A.

II. 1. 2. **2.** a) 432 cm^2; b) $\dfrac{24}{25}$.

III. a) $\mathscr{P}$ = 60 cm; $\mathscr{A} = 108\sqrt{3}$ cm^2; b) $AC = 12\sqrt{3}$ cm; c) $\mathscr{A}_{\Delta AOB} = 48\sqrt{3}$ cm^2; $\mathscr{A}_{\Delta COD} = 12\sqrt{3}$ cm^2.

CAPITOLUL 6
Cercul

6.1. Cercul

3. a) 60°; b) 90°: c) 120°; d) 180°. **4.** a) 10; b) $10\sqrt{2}$; c) 20; d) $10\sqrt{3}$. **5.** a) $4\sqrt{3}$; b) $4\sqrt{2}$; c) 0. **6.** a) 12; b) $6\sqrt{2}$; c) $4\sqrt{3}$. **7.** R = 6 cm; $AB = 6\sqrt{3}$. **8.** a) 120° sau 60°; b) 90°; c) 30° sau 150°. **9.** 12 cm. **10.** $\mathscr{A} = 162\sqrt{3}$ cm^2; PN = 27 cm. **11.** a) $\mathscr{A} = 75\sqrt{3}$ cm^2; $\mathscr{P}$= 50 cm; b) $\mathscr{A} = 50\sqrt{3}$ cm^2; $\mathscr{P} = 10\left(3+\sqrt{3}\right)$ cm. **12.** 75 cm^2. **13.** a) $5\left(\sqrt{6}+\sqrt{2}\right)$; b) 25 cm^2. **14.** $\mathscr{A}$= 128 cm^2; $\mathscr{P} = 32\sqrt{2}$ cm.

6.2. Unghi înscris în cerc

1. a) 90°; 130°; 90°; 50°; b) 30°; 60°; 20°; 70°. **2.** 60°; 65°; 55°. **3.** 100°; 160°; 100°. **4.** a) 24 cm;

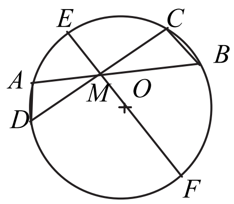
b) 16 cm^2. **5.** 130°; 125°; 105°. **6.** 200°; 60°; 100°. **7.** a) $\mathscr{P}$ = 56 cm; $\mathscr{A}$ = 192 cm^2; b) 9,6 cm; c) 138 cm^2. **8.** a) m($\sphericalangle BAD$) + m($\sphericalangle BCD$) = 360° : 2 = 180°; b) m($\sphericalangle BDC$) = m($\sphericalangle BAC$) = m$(\overset{\frown}{BC})$: 2. **11.** a) $\mathscr{P}$ = 60 cm; $\mathscr{A}$ = $108\sqrt{3}$ cm^2; b) $8\left(3+2\sqrt{3}\right)$ cm. **12.** a) 60°; b) 110°. **13.** 40°. **14.** 45°. **16.** a) *BCEF* inscriptibil; b) *AEHF* inscriptibil.

6.3. Poziţiile relative ale unei drepte faţă de un cerc

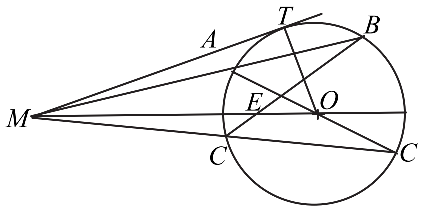
1. a) *a* ext. cercului; b) secantă; c) tangentă; d) secantă. **2.** $AB = 9\sqrt{3}$ cm. **3.** a) concentrice; d) secante; f) exterioare. **4.** a) $\Delta MOA \equiv \Delta MOB$; b) *MQ* mediatoarea lui [*AB*]. **5.** a) 28 cm; b) 48 cm^2. **6.** $\mathscr{P}$ = 40 cm; $\mathscr{A}$ = $20\sqrt{21}$ cm^2. **7.** 1 cm. **8.** 8 cm. **9.** 20 cm; $\sqrt{1714}$ cm. **10.** a) $BH \parallel A'C$; $CH \parallel A'B$; b) $HA' \cap BC = \{N\}$, *N* – mijlocul lui [*BC*]. **11.** a) $\sphericalangle ABC \equiv \sphericalangle ABD$; b) $6\sqrt{6}$ cm. **12.** $\mathscr{P}$ = 11 cm. **13.** m($\sphericalangle BAC$) = 50°. **14.** R = 7,5 cm. **15.** R = 6,25 cm. **16.** r = 1,5 cm. **17.** r = = 7,2 cm.

6.4. Probleme pentru olimpiade şi concursuri

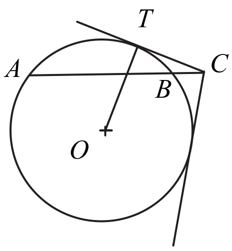
1. $\Delta MAD \sim \Delta MCB \Rightarrow \dfrac{MA}{MC} = \dfrac{MD}{MB} \Rightarrow MA \cdot MB = MC \cdot MD \Rightarrow q = MA \cdot MB = ME \cdot MF =$ $= (OE - OM)(OF + OM) = (R - d)(R + d) = R^2 - d^2 = d$. **2.** $\Delta MAD \sim \Delta MCB \Rightarrow \dfrac{MA}{MC} = \dfrac{MD}{MB} \Rightarrow$ $\Rightarrow MA \cdot MB = MC \cdot MD \Rightarrow q = MA \cdot MB = ME \cdot MF = (MO - OE)(MO + OF) = (d - R)(d + R) =$ $= d^2 - R^2 = MT^2$. **3.** Conform puterii punctului *C* față de cerc, avem $CT^2 = CB \cdot CA \Rightarrow$ $\Rightarrow CT = R\sqrt{3}$.



Problema 1



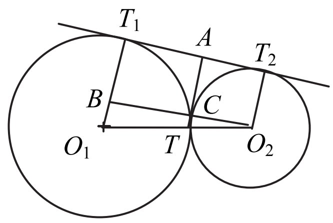
Problema 2



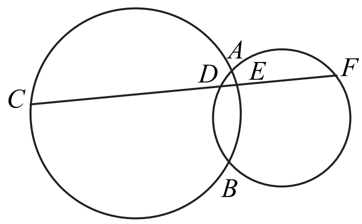
Problema 3

4. Fie $O_2B \perp O_1T_1$, $TA \perp T_1T_2$, $TA = d$, $O_1T_1 = Y_1$, $O_2T_2 = Y_2$, $O_2T \cap AT = \{C\}$. Din $\dfrac{CT}{O_1B} = \dfrac{TO_2}{O_1O_2} \Rightarrow$

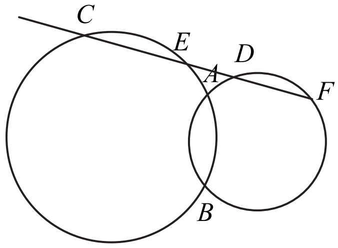
$$\Rightarrow \frac{d - Y_2}{Y_1 - Y_2} = \frac{Y_2}{Y_1 + Y_2} \Rightarrow \frac{2}{d} = \frac{1}{Y_1} + \frac{1}{Y_2}.$$



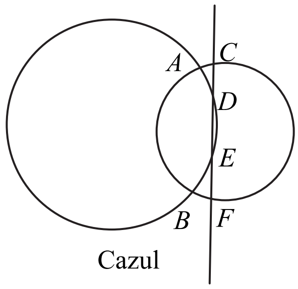
5. În cazul I avem $\sphericalangle CAD = \sphericalangle ADF - \sphericalangle$ACE $= \sphericalangle ABF - \sphericalangle ABE = \sphericalangle EBE$. Analog avem în cazul II. În cazul al III-lea unghiurile $\sphericalangle CAD$ şi $\sphericalangle EBF$ sunt suplementare.



Cazul I

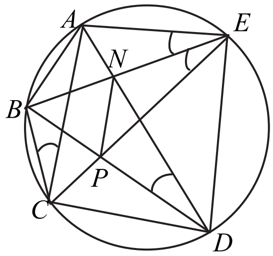


Cazul II

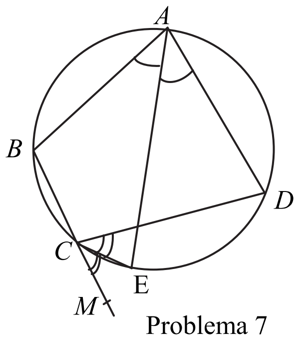


Cazul

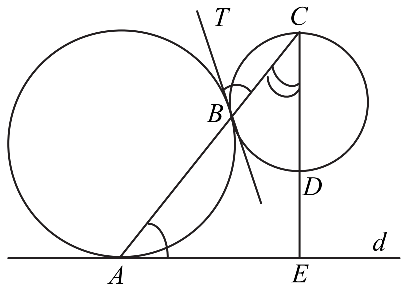
6. $AB \equiv BC \Rightarrow$ m($\sphericalangle ADB$) = m($\sphericalangle BEA$) = m($\sphericalangle BEC$) = m($\sphericalangle BCA$). Patrulaterele $DENP$, $ACDE$ sunt inscriptibile şi rezultă m($\sphericalangle ACE$) = m($\sphericalangle ADE$) = m($\sphericalangle NDE$) = m($\sphericalangle NPE$) $\Rightarrow NP \parallel AC$. **7.** Avem m($\sphericalangle BAD$) + m($\sphericalangle BCD$) = 180°. Fie ($AE$ şi (CE bisectoarele unghiurilor $\sphericalangle BAD$ şi $\sphericalangle MCD$. Avem m($\sphericalangle MCE$) = $\frac{1}{2}$(180° – m($\sphericalangle BCD$)) = $\frac{1}{2}$m($\sphericalangle BAD$) = m($\sphericalangle BAE$) $\Rightarrow$ $ABCE$ inscriptibil. **8.** Fie BT tangentă comună. Avem $\sphericalangle FAB \equiv \sphericalangle ABE \equiv \sphericalangle CBT$. Cum $\sphericalangle BCD$ şi $\sphericalangle CTB$ sunt complementare, rezultă că $\sphericalangle EAC$ şi $\sphericalangle ACE$ sunt complementare şi deci $CE \perp d$. Cum $CB \perp BD \Rightarrow ABDE$ inscriptibil.



Problema 6

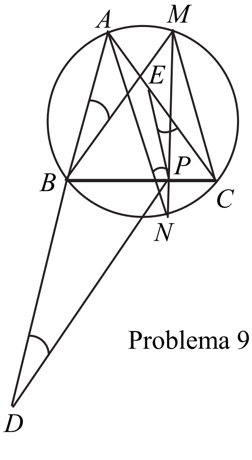


Problema 7



Problema 8

9. Avem $\sphericalangle ADP \equiv \sphericalangle ABM \equiv \sphericalangle ANM$. Punctele A, D, N, P sunt conciclice (pe un cerc $\mathcal{C}$).
Din $\sphericalangle ANP = \sphericalangle ACM = \sphericalangle PEC \Rightarrow E \in \Gamma$.



Problema 9

11. Din $\Delta ADE \sim \Delta DBE$ şi $\Delta ACE \sim \Delta BDE \Rightarrow$

$\Rightarrow \frac{AD}{BC} = \frac{DE}{BE}; \frac{AC}{BD} = \frac{AE}{BE} \Rightarrow$ relația dată.

12. Avem $\sphericalangle BDF \equiv \sphericalangle BAD \equiv \sphericalangle DAC \Rightarrow$

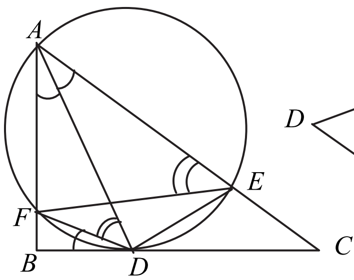
$\Rightarrow \sphericalangle AEF = \sphericalangle ADF = \sphericalangle ADB - \sphericalangle BDF =$

$= C + \frac{A}{2} - \frac{A}{2} = C \Rightarrow EF \parallel BC.$

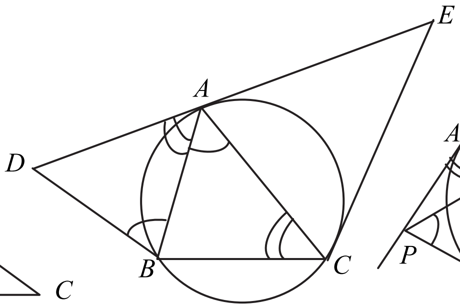
Cum $\sphericalangle BAD \equiv \sphericalangle CAD \Rightarrow DE = DF$.

13. Din asemănarea triunghiurilor ΔACB şi ΔBAD rezultă $\frac{BC}{AD} = \frac{AC}{AB} = \frac{AB}{BD}$ şi $\frac{BC}{AE} = \frac{AC}{EC} = \frac{AB}{AC}$.

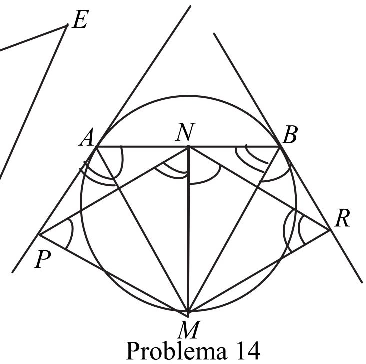
Atunci $\frac{BC}{AD}\cdot\frac{BC}{AE}=\frac{AC}{AB}\cdot\frac{AB}{AC}=1\Rightarrow BC^2=AD\cdot AE$; $\frac{AC}{EC}\cdot\frac{DB}{AB}=\left(\frac{AB}{AC}\right)^2\Rightarrow\frac{DB}{EC}=\left(\frac{AB}{AC}\right)^3$.



Problema 12



Problema 13



Problema 14

14. Din patrulaterele inscriptibile $MNAP$ şi $MNBR$ avem $\sphericalangle MPN=\sphericalangle MAN=\sphericalangle MBR=\sphericalangle MNR$, $\sphericalangle MNP\equiv\sphericalangle MAP\equiv\sphericalangle MBA\equiv\sphericalangle MRN\Rightarrow\Delta MNP\sim\Delta MRN\Rightarrow\frac{MN}{MR}=\frac{MP}{MN}\Rightarrow MN^2=MP\cdot MR$.

15. $AD=\frac{\left|b^2-c^2\right|}{a}$.

TESTE DE EVALUARE

Testul 1

1. $MO\perp AB$: $NO\perp AB\Rightarrow M, O, N$ coliniare. **2.** $m(\sphericalangle A)=75°$; $m(\sphericalangle B)=90°$; $m(\sphericalangle C)=105°$; $m(\sphericalangle D)=90°$. **3.** $\mathscr{A}_{\Delta ABC}=216\text{ cm}^2$; $\mathscr{P}_{\Delta ABC}=72$ cm. **4.** $AB=18\sqrt{2}\left(\sqrt{3}+1\right)$ cm.

Testul 2

1. 3 cm. **2.** $\mathscr{A}=80\sqrt{21}\text{ cm}^2$. **3.** $\mathscr{P}=10\left(3+\sqrt{34}\right)$ cm; $\mathscr{A}=375\text{ cm}^2$. **4.** $R=\frac{35\sqrt{6}}{24}$ cm; $r=$ $=\frac{2\sqrt{6}}{3}$ cm.

VARIANTE DE LUCRĂRI SEMESTRIALE
Semestrul al II-lea

Varianta 1

I. 1. a) 2; b) 8. **2.** a) 5; b) 34. **3.** a) 25 cm; b) $64\sqrt{3}$ cm.
II. 1. 8. **2.** $x \le 1$. **3.** 400 km
III. 1. a) $\mathscr{P}$= 80 cm; b) 9,6 cm; c) *MNPQ* este paralelogram cu laturile paralele cu diagonalele rombului.

Varianta 2

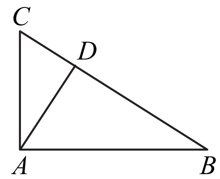
I. 1. a) 3; b) $\frac{1}{5}$. **2.** a) 1; b) $2\sqrt{2}-1$. **3.** a) 32 cm; b) 34 cm.
II. 1. 0. **2.** a) 18, respectiv 12; b) 60%.
III. 1. a) $\mathscr{A} = 256\sqrt{3}$ cm^2; b) $AC = BD = 8\sqrt{19}$ cm; c) d(O, AB) = $5\sqrt{3}$ cm; d(O, CD) = = $3\sqrt{3}$ cm.

Varianta 3

I. 1. a) $-8x$; *b*) 3. **2.** a) –36; b) $\frac{2}{5}$. **3.** a) P = 40 cm; b) $4\sqrt{13}$ cm.
II. 4. a) 21; b) $(x-2)(x+2)(y-2)(y+2)$. **5.** a) $78\sqrt{3}$ cm^2; b) $8\sqrt{3}$ cm.

Varianta 4

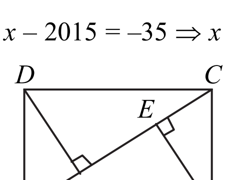
I. 1. $2\sqrt{3}$. **2.** $(9x-5)(9x+5)$. **3.** $x \in \{-2; -1; 0; 1; 2; 3; 4\}$. **4.** $\sqrt{26}$. **5.** 0. **6.** 14 cm.
II. 1. a) $(3x-2)(x^2-x+4)-(x+1)(3x^2-2x+6) = 3x^3-3x^2+12x-2x^2+2x-8-3x^3+2x^2-$ $-6x-3x^2+2x-6 = -6x^2+10x-14$.



2. Notăm $BD = x \Rightarrow CD = 2x \overset{\text{T.înălțimii}}{\Rightarrow} 2x^2 = 64 \cdot 2 \Rightarrow x = 8 \Rightarrow BD = 8$;
$CD = 16$ cm; $BC = 8 + 16 = 24$ cm $\Rightarrow AB = 8\sqrt{3}$ cm;
$AC = \sqrt{256+128} = \sqrt{384} = 8\sqrt{6}$ cm;
$\mathscr{P}_{\Delta ABC} = 24 + 8\sqrt{3}+8\sqrt{6} = 8\left(3+\sqrt{3}+\sqrt{6}\right)$ cm.

Varianta 5

I. 1. x^2-5x+8. **2.** $\frac{25}{8}$. **3.** $\sqrt{7}$. **4.** b) (–1; –1). **5.** $\left(\sqrt{5}\right)^2$; $\sqrt{25}$. **6.** $324\sqrt{3}$ cm^2.
II. 1. $10 + 20 - |x-2015| = -5 \Leftrightarrow |x-2015| = 35 \Rightarrow x-2015 = 35$ sau $x-2015 = -35 \Rightarrow x =$ $= 2050$ sau $x = 1980$; $S = \{1980; 2050\}$.



2. Fie $BE \perp AC$, $E \in AC$, $AF \perp AC$, $F \in AC$; $AC^2 \Rightarrow AC = 25$;
$BC^2 = EC \cdot AC \Rightarrow EC = \frac{225}{25} = 9$ cm $\Rightarrow EF = 7$ cm.

TESTE FINALE

TESTUL 1

I. 1. 25. **2.** 120°. **3.** 4. **4.** 9 cm. **5.** 1. **6.** 14.

II. 2. $(3x+2)^2 = 9x^2 + 12x + 4$; $(2x-1)^2 = 4x^2 - 4x + 1$; $5x^2 + 16x + 3$. **3.** $A + b = 14$; $a - b = 4 \Rightarrow$ $\Rightarrow 2a = 18 \Rightarrow a = 9$; $b = 5$. **4.** $\sqrt{\left(\sqrt{3}-2\right)^2} = \left|\sqrt{3}-2\right| = 2-\sqrt{3}$; $\sqrt{\left(\sqrt{3}-3\right)^2} = \left|\sqrt{3}-3\right| = 3-\sqrt{3}$.

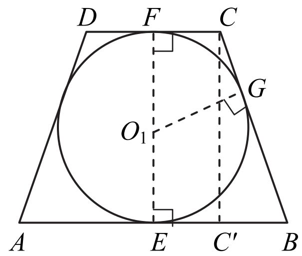
III. 1. $AD \parallel CN \Rightarrow \frac{CN}{AD} = \frac{MC}{MD} \Rightarrow \frac{CN}{15} = \frac{1}{3} \Rightarrow CN = 5$ cm $\Rightarrow BN = 20$ cm; $BC \parallel PD \Rightarrow$ $\Rightarrow \frac{MC}{MD} = \frac{BC}{PD} \Rightarrow \frac{1}{3} = \frac{15}{PD} \Rightarrow PD = 45$ cm $\Rightarrow AP = 60$ cm.

2. a) Fie MN linie mijlocie a trapezului $ABCD \Rightarrow$

$\Rightarrow MN = \frac{B+b}{2} \Rightarrow MN = \frac{32+10}{2} = 21$ cm;

$[BE] \equiv [BG]$ şi $[CF] \equiv [CG] \Rightarrow BC = 16 + 5 = 21$ cm.

În $\Delta AC'C$, m($\sphericalangle C'$) = 90° avem $AC^2 = AC'^2 + CC'^2 \Rightarrow AC^2 =$ $= 21^2 + \left(8\sqrt{5}\right)^2 \Rightarrow AC = 31$ cm; $BD = 31$ cm;



b) $\mathscr{A}_{ABCD} = \frac{(B+b)\cdot h}{2} = \frac{(32+10)\cdot 8\sqrt{5}}{2} = 168\sqrt{5}$ cm^2;

$\mathscr{P}_{ABCD} = 32 + 10 + 21 \cdot 2 = 84$ cm.

TESTUL 2

I. 1. $\frac{9}{16}$. **2.** $\frac{3\sqrt{2}-4\sqrt{3}}{2}$. **3.** 3. **4.** 24. **5.** 12π. **6.** 7,57.

II. 2. a) –4; b) $\frac{8}{45}$. **3.** $p = \frac{1}{2}$; $p = \frac{7}{36}$. **4.** $(x + 1)$.

III. 1. Fie $T \in (BC)$, $ET \parallel FQ$, FQ linie mijlocie în $\Delta BET \Rightarrow [BQ] \equiv [QT]$; ET linie mijlocie în $\Delta CQA \Rightarrow [QT] \equiv [TC] \Rightarrow BQ = \frac{1}{3}BC$. **2.** $R = 8$ cm; $l = 8\sqrt{2}$ cm; $\mathscr{A} = 128$ cm^2.

TESTUL 3

I. 1. 1. **2.** 1. **3.** 1. **4.** 1. **5.** $\frac{3}{8}$. **6.** 7,32. **7.** 12. **8.** $5\sqrt{2}$. **9.** 36. **10.** 300.

II. 1. a) $a = \frac{25}{36}\cdot 36 - 5 + \frac{4}{100}\cdot 100 = 24$; $b = \sqrt{9} + \sqrt{64} - \sqrt{25} = 6$; $m_g = \sqrt{ab} = 12$;

b) $\left(2\sqrt{3}-\sqrt{2}\right)^2 + \left(2\sqrt{3}+\sqrt{2}\right)^2 - \left(3\sqrt{3}-\sqrt{5}\right)\left(\sqrt{27}+\sqrt{5}\right) = 12 - 4\sqrt{6} + 2 + 12 + 4\sqrt{6} + 2 - 27 + 5 =$ $= 6$; c) $12(2x + 3) - 10(x + 5) = 20(x + 2) - 15(3x + 1) \Leftrightarrow 14x - 14 = -25x + 25 \Leftrightarrow x = 1$.

2. a) $AO = 12 \Rightarrow AC = 24$ cm $\Rightarrow \mathscr{A}_{ABCD} = AD \cdot AC = 384$ cm^2; b) $BD = 40$ cm; c) $AB = 8\sqrt{13}$; $\mathscr{P}_{ABCD} = 32 + 16\sqrt{3}$; $32 + 16\sqrt{3} < 28 + 32 \Leftrightarrow 16\sqrt{3} < 28 \Leftrightarrow 768 < 78$.

TESTUL 4

I. 1. –2. **2.** $\frac{3}{2}$. **3.** 9. **4.** 10. **5.** $\frac{3}{50}$. **6.** 7. **7.** 36. **8.** 3. **9.** $\frac{4}{5}$. **10.** 12.

II. 1. a) $|3x-1|-3=\pm 2$; 1° $|3x-1|=1 \Rightarrow 3x-1=\pm 1 \Rightarrow x=\frac{2}{3}$ sau $x=0$; 2° $|3x-1|=5 \Rightarrow$ $\Rightarrow 3x-1=\pm 5 \Rightarrow x=2$ sau $x=-\frac{4}{3}$; b) $\left(3\sqrt{2}-2\right)^2+4\left(3\sqrt{2}-2\right)+4=22-8+4=18$; c) $a=$ $=3-\sqrt{5}$; $b=3+\sqrt{5}$; $m_a=3$; $m_g=2$. **2.** a) $AC=\sqrt{192+64}=16$ m; b) $\mathscr{P}_{ABCD}=$ $=2\left(8\sqrt{3}+8\right)=16\sqrt{3}+16$; $16\sqrt{3}+16<44 \Leftrightarrow 16\sqrt{3}<28 \Leftrightarrow 768<784$; c) $BE=4\sqrt{3}$; $EC=4$; $FE=6\sqrt{3}$; $10\sqrt{3}<18$.

TESTUL 5

I. 1. $\frac{14}{9}$. **2.** 1. **3.** –4. **4.** $16k^2$. **5.** 53. **6.** $x=2$; $y=3$. **7.** $15\sqrt{3}$. **8.** $10\sqrt{10}$. **9.** 9. **10.** 15.

II. 1. a) 6; b) 0; c) $x=1$. **2.** a) Fie trapezul $ABCD$, $AB \parallel CD$, $AB=AD=BC$. Dacă $[MN]$ este linia mijlocie a trapezului şi $[PQ]$ este segmentul determinat pe linia mijlocie a trapezului de către diagonalele acestuia, avem $PQ=\frac{1}{2}(DC-AB)$; $PQ=\frac{1}{3}MN$; $MN=\frac{1}{2}(DC+AB) \Rightarrow$ $\Rightarrow 4AB=2DC \Rightarrow AB=10$ cm şi $\mathscr{P}_{ABCD}=50$ cm. Înălţimea trapezului este $5\sqrt{3}$ cm $\Rightarrow \mathscr{A}_{ABCD}=$ $=75\sqrt{3}$ cm^2; b) Fie $BE \perp DC$, $E \in DC$. În ΔBEC, m($\sphericalangle E$) = 90°, $BC=10$ cm, $EC=5$ cm $\Rightarrow$ $\Rightarrow$ m($\sphericalangle EBC$) = 30° $\Rightarrow$ m($\sphericalangle ABC$) = 120°. Aşadar m($\sphericalangle DAB$) = m($\sphericalangle BAC$) = 120°, m($\sphericalangle ADC$) = = m($\sphericalangle BCD$) = 60°; c) ΔABC este isoscel cu m($\sphericalangle ABC$) = 120° $\Rightarrow$ m($\sphericalangle BAC$) = m($\sphericalangle BCA$) = 30°, apoi m($\sphericalangle ACD$) = 30° $\Rightarrow$ $[CA$ este bisectoarea unghiului $\sphericalangle BCD$. Analog $[DB$ este bisectoarea unghiului $\sphericalangle ADC$. În $\Delta ADC \Rightarrow$ m($\sphericalangle DAC$) = 90° $\Rightarrow CA \perp AD$. Analog $DB \perp BC$.

TESTUL 6

I. 1. 0. **2.** 1. **3.** $\{0, 1, 2, 3\}$. **4.** $x=\frac{19}{2}$; $x=11$. **5.** –1. **6.** $x \in \left\{-\frac{11}{2}, -3, -\frac{3}{2}, -1, 0, \frac{1}{2}, 2, \frac{9}{2}\right\}$. **7.** $20\sqrt{3}$. **8.** $30\sqrt{3}$. **9.** $40\sqrt{2}$. **10.** $\frac{3}{5}$.

II. 1. a) $a=\frac{30}{100}b=\frac{3}{10}b \Rightarrow a+b=\frac{13}{10}b=130 \Rightarrow b=100$; $a=30$; b) $a^2+b^2-2ab=(a-b)^2 \geq$ ≥ 0, $\forall\, a, b \in \mathbb{R} \Leftrightarrow a^2+b^2 \geq 2ab$, $\forall\, a, b \in \mathbb{R}$; c) Conform b) avem $x^2+y^2 \geq 2xy$; $x^2+z^2 \geq 2xz$; $y^2+z^2 \geq 2yz$. Adunând obţinem $2x^2+2y^2+2z^2 \geq 2xy+2xz+2yz \mid :2 \Rightarrow x^2+y^2+z^2 \geq xy+$ $+xz+yz$, $\forall\, x, y, z \in \mathbb{R}$. **2.** a) Din m($\sphericalangle A$) + m($\sphericalangle B$) = 180° şi m($\sphericalangle B$) = 2m($\sphericalangle A$) $\Rightarrow$ m($\sphericalangle A$) = = m($\sphericalangle C$) = 60°; m($\sphericalangle B$) = m($\sphericalangle D$) = 120°; b) ΔABD este dreptunghic în B. Din m($\sphericalangle A$) = 60° şi $AB=12$ cm $\Rightarrow AD=BC=24$ cm, deci $\mathscr{P}_{ABCD}=72$ cm; c) Din ΔABD, m($\sphericalangle B$) = 90° $\Rightarrow BD=$ $=12\sqrt{3}$ şi $\mathscr{A}_{ABCD}=144\sqrt{3}$ cm^2.

TESTUL 7

I. 1. $\frac{2}{3}$. **2.** 0 şi $2\sqrt{5}$. **3.** –12. **4.** 8. **5.** 20. **6. 48**.

II. 1. $a + b = 110$; $\frac{80}{100}a - \frac{20}{100}b = 78 \Leftrightarrow \frac{4a}{5} - \frac{b}{5} = 78 \Leftrightarrow 4a - b = 385 \Leftrightarrow b = 4a - 385$;

$a + 4a - 385 = 110 \Leftrightarrow 5a = 495 \Leftrightarrow a = 99$; $b = 11$. **2.** $\frac{\sqrt{2}x+3}{3} - \frac{x}{\sqrt{2}} + \frac{6x}{\sqrt{18}} = \frac{18-\sqrt{2}x}{6} \Leftrightarrow$

$\Leftrightarrow 2\sqrt{2}x + 6 - 3\sqrt{2}x + 6\sqrt{2}x = 18 - \sqrt{2}x \Leftrightarrow 6\sqrt{2}x = 12 \Leftrightarrow x = \frac{12}{6\sqrt{2}} \Leftrightarrow x = \sqrt{2}$.

3. $x = (a^2 - 3a)(a^2 - 3a + 8) + 16 \Leftrightarrow x = (a^2 - 3a + 4)^2$ este pătrat perfect.

4. a) În ΔOMA, $m(\sphericalangle OMA) = 90°$, avem $OM^2 = OA^2 - MA^2 \Leftrightarrow OM^2 = 40^2 - 32^2 \Leftrightarrow OM = 24$ cm;

b) $\Delta OMA \equiv \Delta ONA \Rightarrow \mathscr{A}_{AMON} = 2\mathscr{A}_{OMA} = 2 \cdot \frac{OM \cdot MA}{2} = 24 \cdot 32 = 768$ cm^2; c) $MN \perp OA \Rightarrow$

$\Rightarrow MN = 2MP$, unde $\{P\} = AO \cap MN$, deoarece în ΔAMN isoscel, AP este înălţime şi mediană;

$MP = \frac{MO \cdot MA}{OA} = \frac{24 \cdot 32}{40} = \frac{96}{5}$ cm $\Rightarrow MN = \frac{192}{5} = 38{,}4$ cm.

TESTUL 8

I. 1. –40. **2.** 1. **3.** 11. **4.** 12. **5.** 12. **6.** 8.

II. 1. a) $4x^2 - 4x + 1 - 2x - 8 = 4x^2 + 4x + 1 + 3x + 6 - 1 \Leftrightarrow -13x = 13 \Leftrightarrow x = -1$;

b) $x \cdot |1-\sqrt{3}| + |2-\sqrt{3}| = |2\sqrt{3}-3| + 2 \Leftrightarrow x(\sqrt{3}-1) = 3\sqrt{3} - 3 \Leftrightarrow x = 1$. **2.** $E_1 = \frac{1}{4} + \frac{1}{4} + \frac{3}{4} \Leftrightarrow$

$\Leftrightarrow E_1 = \frac{5}{4}$; $E_2 = \left(-\frac{2}{5}\right) : \left(-\frac{2}{5}\right) + 1 \Leftrightarrow E_2 = 2$; $E = E_1 : E_2 = \frac{5}{4} : 2 = \frac{5}{8}$. **4.** a) $AE \parallel DC$ şi $AD \parallel CE \Rightarrow$

$\Rightarrow AECD$ paralelogram; b) Fie $DM \perp AB$, $M \in AB$; $\mathscr{A}_{AECD} = AE \cdot DM \Leftrightarrow 60 = AE \cdot DM \Leftrightarrow$

$\Leftrightarrow 60 = 6 \cdot DM \Leftrightarrow DM = 10$ cm; c) $\mathscr{A}_{CEB} = \frac{BE \cdot d(C, BE)}{2}$; $EB = \frac{3AE}{2} = \frac{3 \cdot 6}{2} = 9$ cm; $\mathscr{A}_{CEB} =$

$= \frac{9 \cdot 10}{2} = 45$ cm^2.

TESTUL 9

I. 1. 26. **2.** 11. **3.** $12\sqrt{2}$. **4.** 300. **5.** $\frac{2}{15}$. **6.** 40.

II. 2. a) x = lungimea drumului; I zi: $\frac{2x}{5} + 10$; II zi: $\frac{3x}{50} + 12 \Rightarrow \frac{2x}{5} + 10 + \frac{3x}{50} + 12 + 140 \Rightarrow$

$\Rightarrow x = 300$ (km). **3.** a) a = numărul apartamentelor cu 2 camere; b = numărul apartamentelor cu 3 camere; $a + b = 50$ şi $2a + 3b = 130 \Rightarrow a = 20$; $b = 30$; b) $\frac{p}{100} \cdot 50 = 20 \Rightarrow p\% = 40\%$.

4. $\frac{|\sqrt{2}+1| + |\sqrt{3}-\sqrt{2}| + |4-\sqrt{3}|}{x-2} \in \mathbb{Z} \Rightarrow \frac{\sqrt{2}+1+\sqrt{3}-\sqrt{2}+4-\sqrt{3}}{x-2} \in \mathbb{Z} \Rightarrow \frac{5}{x-2} \in \mathbb{Z} \Rightarrow$

$\Rightarrow x - 2 \in D_5 \Rightarrow x - 2 \in \{-5, -1, 1, 5\} \Rightarrow A = \{3; 1; 3; 7\}$; $|2x - 1| \leq 3$, $x \in \mathbb{Z} \Rightarrow 2x - 1 \in \{-3, -2, -1, 0, 1, 2, 3\} \Rightarrow 2x \in \{-2, -1, 0, 1, 2, 3, 4\} \Rightarrow x \in \{-1, 0, 1, 2\} \Rightarrow B = \{-1; 0; 1; 2\}$;

$A \cap B = \{1\}$. **5.** Notăm $n^2 + n = a \Rightarrow (a-4)(a-2) + 1 = a^2 - 6a + 9 = (a-3)^2 \Rightarrow E(n) = (n^2 + n - 3)^2$ este p.p.

III. 1. a) $\mathscr{A}_{\Delta CMN} = 100 \Rightarrow \dfrac{MN \cdot BC}{2} = 100 \Rightarrow MN \cdot BC = 200 \Rightarrow 3MN \cdot BC = 600 \Rightarrow AB \cdot BC = 600 \Rightarrow \mathscr{A}_{ABCD} = 600$ (cm^2); b) $MN \cdot 20 = 200 \Rightarrow MN = 10 \Rightarrow AB = 30$ (cm); $AC = 10\sqrt{13}$ cm; c) $\Delta DQC \sim \Delta NQA$. Fie $QE \perp DC$, $E \in (DC)$ şi $QF \perp AB$, $F \in (AB) \Rightarrow \dfrac{DC}{AN} = \dfrac{QE}{QF} \Rightarrow \dfrac{QE}{QF} = \dfrac{3}{2} \Rightarrow$

$\Rightarrow \dfrac{QE}{QE+QF} = \dfrac{3}{5} \Rightarrow \dfrac{QE}{20} = \dfrac{3}{5} \Rightarrow QE = 12$ (cm); $\mathscr{A}_{\Delta DQC} = \dfrac{DC \cdot QE}{2} = \dfrac{30 \cdot 12}{2} = 180$ (cm^2).

2. a) $L_{\text{cerc}} = 2\pi R = 12\pi$ (cm); b) $m(\overset{\frown}{AB}) = 90° \Rightarrow m(\sphericalangle AOB) = 90° \Rightarrow \mathscr{A}_{\Delta AOB} = \dfrac{AO \cdot OB}{2} = 18$ cm^2; c) $\mathscr{A} = \dfrac{\pi R^2}{4} - \mathscr{A}_{\Delta AOB} = 9\pi - 18 = 9(\pi - 2)$ (cm).

TESTUL 10

I. 1. 30. **2.** 360°. **3.** $18\sqrt{2}$. **4.** 27. **5.** 3. **6.** 7,56.

II. 2. $N = \dfrac{1}{4} - \sqrt{3} + 3 + \dfrac{5}{4} + \sqrt{5} + 1 + \dfrac{1}{4} - \dfrac{2\left(\sqrt{5}-\sqrt{3}\right)}{5-3} - \dfrac{2-3}{4}$; $N = 6 \in \mathbb{Z}$. **3.** $4x - 3 - 2(2x + 3) = 5(x - 1) + 5 \Leftrightarrow 4x - 3 - 4x - 6 = 5x - 5 + 5 \Leftrightarrow x = -\dfrac{9}{5}$. **4.** a) $\{a, b, c\}$ d.p. $\{12, 13, 15\}$ şi $a + b + c = 120 \Rightarrow a = 12k$; $b = 13k$; $c = 15k \Rightarrow 12k + 13k + 15k = 120 \Rightarrow k = 3 \Rightarrow$ elevii au rezolvat 36, 39, respectiv 45 de probleme; b) $\dfrac{p}{100} \cdot 45 = 36 \Rightarrow p\% = 80\%$. **5.** $E(x) = x^2 - 2x + 1 + x^2 - 4 + 9 - 6x + x^2 - (4x^2 - 12x + 9) + 2x + 10$; $E(x) = -x^2 + 6x + 7$.

III. 1. a) Fie $CE \perp AB$, $E \in (AB) \Rightarrow CE = AD = 4$ cm; $EB = AB - AE = 10 - 6 = 4$ cm. În ΔCEB, $m(\sphericalangle E) = 90° \overset{\text{T.P.}}{\Rightarrow} CB = 4\sqrt{2}$ cm; $\mathscr{P}_{ABCD} = 20 + 4\sqrt{2}$ cm; b) Fie $MN \perp AD$, $N \in (AD) \Rightarrow MN$ linie mijlocie în trapez $\Rightarrow MN = 8$ cm $\Rightarrow \mathscr{A}_{MAD} = \dfrac{AD \cdot MN}{2} = 16$ (cm^2); c) Fie $DF \perp BC$, $F \in (BC) \Rightarrow \sphericalangle FCD \equiv \sphericalangle CBA$ (corespondente); ΔCEB dreptunghic isoscel $\Rightarrow m(\sphericalangle ABC) = 45° \Rightarrow m(\sphericalangle DCF) = 45° \Rightarrow \Delta DFC$ dreptunghic isoscel $\Rightarrow DF = 3\sqrt{2}$ (cm). **2.** a) $L_{\text{cerc}} = 2\pi R = 24\pi$ (cm); b) $m(\overset{\frown}{AB}) + m(\overset{\frown}{BC}) = 180° \Rightarrow AC$ diametru $\Rightarrow \Delta ABC$ dreptunghic în B, $m(\sphericalangle C) = 30° \Rightarrow AB = 12$ cm şi $AC = 12\sqrt{3}$ cm; $\mathscr{A}_{\Delta ABC} = 72\sqrt{3}$ cm^2; c) $ABCD$ dreptunghi.

Cuprins